

ARMY RESEARCH LABORATORY



Calibration of Inertial Sensors

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ARL-MR-481

OCTOBER 2000

Army Research Laboratory
Aberdeen Proving Ground, MD 21005-5066

ARL-MR-481

October 2000

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Abstract

The calibration of accelerometers, angular rate sensors, and inertial measurement units (IMU) increases the accuracy of their measurements. For artillery rounds and most rockets, measurements are acquired while the body is spinning. By the comparison of the sensor output to known input, calibration is accomplished. Calibration requires a model of the sensor. The model represents the measurement process. Model selection, calibration equipment, and data selection affect the usefulness of a calibration for a given mission. Calibration is model fitting for a specific mission or class of missions.

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CALIBRATION OF INERTIAL SENSORS

1. Introduction

The calibration of accelerometers, angular rate sensors, and inertial measurement units (IMU) increases the accuracy of their measurements. For artillery rounds and most rockets, measurements are acquired while the body is spinning. By the comparison of the sensor output to known input, calibration is accomplished. Calibration requires a model of the sensor. The model represents the measurement process. By starting with complete models and eliminating nonrelevant terms, we can tailor general models for specific devices. With a model, observations are acquired to estimate the unknown model parameters. Response surface techniques, experimental design methods, and regression techniques are used to establish the relationship between the unknown model parameters and the observations. With these methods, it is possible to obtain an expected error structure for a given set of observations; thus, observation sets can be compared to obtain an optimal or nearly optimal set of observations for the calibration of a sensor operating in a given environment. The conditions associated with the launch and flight of artillery and rockets constitute the environment of interest. This report gathers some of the models, discusses the estimation methods, and discusses the optimization of observation sets.

2. Background

Sensors usually undergo three phases of testing. Qualification tests ensure that the sensor will be able to meet the customer specifications in production. These are extensive methods and the procedure is done once before production. The qualification tests will identify sensor shortcomings that can be improved through calibration. Acceptance test procedures are performed on each production unit. Calibration is part of the acceptance test. Reliability tests use statistical procedures to quantify the ability of the items to perform as intended.

A large body of work is associated with calibration. In the Institute of Electrical and Electronics Engineers (IEEE) standards section, industrial standards for accelerometers and angular rate sensors are presented. These articles contain models and calibration procedures for specific classes of devices and format guides for specifications. Articles, notes, and view graphs from courses, and books provide information about modeling and calibrating inertial devices. There are numerous journal articles describing the calibration of sensors and IMUs. The sophistication of the model corresponds directly to the measurement

accuracy achieved through calibration. Diminishing gains in measurement accuracy are to be expected as the model increases in complexity. For given applications, different calibration procedures will be appropriate.

Typically used reference signals are gravity for accelerometers and the earth's rotational rate for angular rate sensors. Normally, calibration takes place in laboratories fixed to the earth. For high accuracy applications, the calibration facility needs to be properly designed and equipped. Isolation of the sensor from vibration and the ability to determine the orientation of a sensor constitute major concerns in laboratory design. The direction of true north must be known so that effects attributable to earth rotation can be controlled. Given suitable reference signals, it is sometimes possible to calibrate a device while it is operating. Kalman filtering can be used for in-flight calibration via known reference signals; for example, a global positioning system (GPS) receiver can provide useful information.

After a calibration model has been chosen, a suitable set of observations needs to be collected. The set of observations must contain values that allow the calibration parameters to be estimated. Linear parameters require at least two distinct values, while quadratic forms require three values. Repeated measurements are desirable as they allow the estimation of the calibration error. In many cases, changing the orientation of the sensor to the reference signal generates the observation set. An accurate method to accomplish this is required. Reference signals can be generated by rate tables or flight simulators.

Devices should be calibrated over the intended range of use. The error associated with the calibration process will grow exponentially outside the calibration range. Using the calibration model outside the range used for calibration is extrapolation. If proper techniques have been employed, it is possible to predict the error associated with the model for a specific operating point. The error associated with a given calibrated sensor depends on the observation set, the model used, and the residual error. Methods of least squares can be used to predict error at specific operating points. The purpose of experimental design and response surface methodology is to optimize the set of observations. The optimal set of observations minimizes the calibration error over a specified operating range, assuming a given number of observations.

3. Models

A model is an abstraction of reality to facilitate understanding or to accomplish a specific task. For calibration, a mathematical model is formulated to represent the measurement process. Within the model, known sources of variation are represented. Estimating the values of sources of variation constitutes calibration.

For sensor models, the output is typically a voltage. The dependence of the voltage on the explanatory variables is represented functionally. Generally, the output variable is called the dependent variable and the explanatory variables are called independent variables. The unknown's values in the model, which need to be estimated, are referred to as parameters. Least squares is typically used to estimate model parameters.

For calibration to proceed, a model of the sensor measurement is needed. The model contains and summarizes all the information known about a specific device. Models could be as simple as a scale factor and a bias; however, the accuracy of the calibrated sensor increases with model complexity. The trade-off between complexity and accuracy is fundamental to the goal of calibration. While simple models may apply to a class of devices, the complex models are based on specific information about the sensor. Multiple sensor devices are modeled more succinctly with matrix notation.

Model components depend on the manufacturing process and the way the sensor generates its signal. The IEEE standards are an internationally recognized source of information about calibration models. For a sensor based on new technology, the calibration process may be impeded by the lack of a model. If factory calibration is to be of value, it is important for the engineer to know the calibration model and the set of observations used for parameter estimation. Ideally, the purchaser would specify the calibration model used as part of the acceptance test.

For inertial devices, the designer attempts to provide a linear relationship between the quantity of interest and the output voltage. For the user, it is reasonable to hope that a straight line will explain the variation in the output or dependent variable. When this is done, the y-intercept is referred to as the bias and the slope is called the scale factor. Sometimes, higher order fits are used; in this situation, the scale factor is the higher order model and not a single term. For higher order scale factors, the bias will be the y-intercept of the fit. The linear scale factor is used to denote the linear term, and the quadratic term is referred to as the g-squared effect. In the literature, there is some inconsistency in the use of the term scale factor; the term sometimes refers to the estimated slope of the linear term in the sensor calibration model.

Asymmetry usually refers to different sensor behavior for positive and negative values. In this case, separate fits need to be done for each region. Asymmetry does not have to take place at the zero value. During calibration, it is worthwhile to check for asymmetry. This can be done by curve fitting each portion of the data and then statistically comparing the models of scale factor.

Hysteresis is the separation of the output between ascending and descending input values. During calibration, this can be checked by monotonically increasing the input values and then monotonically decreasing the input values.

Temperature affects the properties of the inertial sensors and can have a dramatic effect on the output. Sophisticated models of temperature exist and have been used with good results. By repeating the calibration process at several temperatures over the expected range of use, it is possible to account for and quantify this variable.

Shock can damage or destroy inertial sensors. The operating characteristics of a device can change because of shock. Sensors can be calibrated, shocked, and then re-calibrated to determine if shock is a potential problem.

Changes in inertial devices because of aging and other effects are lumped into the category of day-to-day errors. Shelf life, atmospheric effects, vibration, and transportation effects all contribute to this error.

Threshold is defined after the scale factor has been modeled. It is defined as the output value increasing from minimum input that results in an output of at least half the expected value. The “dead band” is the interval around the null in which small input changes cannot be detected. The value at which the input becomes readable via the scale factor is called the resolution of the system. The resolution is the minimum measurable input. These values can be ascertained after a scale factor has been accepted.

The most common types of accelerometers are pendulous, vibrating string, and fiber optical. There are several versions of each. General descriptions are given in Lawrence (1998) or Chatfield (1997). Detailed models are described in the IEEE standards. IEEE Standard 836-1991, entitled “IEEE Recommended Practice for Precision Centrifuge Testing of Linear Accelerometers,” contains a detailed model and suggests testing procedures. Single degree-of-freedom gyroscopes measure the rate of change around a single axis. A large body of literature exists for spinning or rotating wheel gyroscopes. These are not pertinent to the artillery and rocket setting. Strap-down devices based on vibration are abundant. Lawrence (1998) and Chatfield (1997) give useful explanations of these devices. The IEEE standards provide calibration guides.

When these devices are combined to form an IMU, new problems arise. Most are related to axis misalignment. Chatfield (1997) gives an example of calibration for three axis accelerometers and three axis gyroscopes. An in-depth knowledge of the relevant standards improves the ability of an organization to calibrate both individual sensors and IMUs.

4. Parameter Estimation

After a model has been chosen, the parameters are estimated by choosing the values that minimize the square of the differences between the model values and the observations. Within the field of statistics, regression analysis contains a large body of theory and techniques for parameter estimation. The set of observations determines the covariance matrix of the parameters. The covariance matrix provides a means to characterize the performance of the sensor at any operating point. Calibration is least squares parameter estimation, and its usefulness will depend on the observations. The precision of the calibration will depend on the model selected, the number of observations, and the location of the observations. A sensor should be calibrated over its operating range. The error will increase with the square of the distance from the centroid of the observations, with the curvature becoming pronounced outside the calibrated range. A model will be more precise over a limited range than over a large range.

If the value of a parameter is not significantly different from zero, it is reasonable to drop that term from the model. In many cases, dropping insignificant terms improves the model's predictions. It is sometimes desirable to compare models. Statistically, models are compared, based on their coefficients of determination or r^2 . This process compares the variation each model accounts for and allows the difference to be statistically quantified.

A set of observations can be evaluated *a priori* based on the structure of parameter's covariance matrix. If X is the matrix where each row is the values of the independent variables for an observation, then the structure of the parameter covariance can be written as $(X X)^{-1}$. With this matrix, it is possible to compare different observation sets. If the columns of the X matrix are similar, it is difficult to distinguish between the two corresponding independent variables; this problem is called colinearity. Colinearity in the X matrix or observation matrix will be imparted to the parameter covariance matrix. Seber (1977) discusses colinearity and regression analysis. A matrix containing severe colinearity is said to be ill conditioned. Colinearity in the X matrix can be reduced by the judicious selection of observations. The IEEE standards recommend that the number of observations be $4(N+2)$, where N is the number of parameters to be estimated; however, examination of the parameter covariance matrix leads to a more efficient recommendation. The instruments available for calibration may restrict the range of observations. The model coefficients will also determine segments of the observation process (for example, to find the quadratic coefficient, at least three different measurements are needed along the scale of interest).

Optimality is usually discussed in terms of the $X'X$ matrix. Ideally, this would be a diagonal matrix with small numbers along the diagonal. The diagonal elements

when multiplied by the residual variance will be the variance of the respective parameters. In the context of calibration, optimality can be defined as the observation set that minimizes the error over a given operating range. Reducing colinearity in the X matrix typically indicates a decrease in parameter variance, thereby reducing the overall model error.

The influence each point has on the parameter values can be calculated. Chatterjee and Hadi (1988) discuss the influence function in detail. Sometimes, a large influence is attributable to a judicious observation, but in other cases, it may be attributable to a large residual. Large residuals can be outliers or points which, because of spurious conditions, do not fall in the same pattern as the other points. Including outliers can increase the error of the calibration process.

When calibration runs are expensive, the techniques of experimental design and response surface modeling can be used to extract the maximum amount of information for a fixed number of observations. Kalman filtering is an application of least squares to dynamic systems. In some cases, filters can be designed to calibrate sensors.

5. Operating Environment

The mission environment of the sensor must be known, within limits, if a sensor is to be properly calibrated. For an artillery shell or rocket, the desired calibration conditions can be difficult to achieve. First, the sensor must be able to undergo a shock as great as 15,000 g's without changing the calibration. During flight, spin rates may exceed 300 Hz. The vibrations can be harsh. Large spin rates around the direction of flight and relatively large drag in the direction of flight through cross-axis coupling can overpower signals along the other dimensions. Pressure waves caused by rocket motors or supersonic flight will travel through the sensor and may affect the sensing element. Currently, the vibration environment of rockets and artillery is not well known. Temperature can change dramatically over the flight. Battery voltage may vary over a flight; thus, sensors should be tested at realistic voltage levels. The effects of voltage could be determined by calibrating at different voltage levels and comparing the models.

With a flight simulator, it is possible to achieve high spin rates. By using the flight simulator as a centrifuge, it may be possible to put high g loads on an accelerometer. Temperature-controlled rate tables can be used to model effects of temperature in low-g environments. Without knowledge of the pressure waves, it is difficult to devise a test and select equipment to allow these effects to be modeled. Sensors can be calibrated, shocked on a shock table, and then re-calibrated to simulate the effects of launch on calibration. After flight tests, if

sensors are recovered, it would be possible to re-calibrate them and compare this model to the pre-flight test calibration.

When individual sensors are combined to make an IMU, it makes sense to calibrate them before the assembly phase. After assembly, calibration would focus on the estimation of misalignment and proximity effects. A two-stage calibration procedure increases system accuracy by minimizing the possibility of confounding error sources.

6. Conclusions

The first step in the calibration process is the selection of a calibration model. This choice affects the accuracy of the calibrated sensor and the amount of information that can be acquired. The IEEE standards include models and calibration methods for many types of sensors. The manufacturer should be able to supply a model. Next, it is necessary to select a set of observations. The observations should cover the range of sensor use and allow the estimation of each model parameter. The first time a sensor is used, it is prudent to check for scale asymmetry, temperature effects, and other potential problems.

Details associated with the calibration of individual devices for specific missions have not been addressed. Instead, the issues and procedures for calibrating inertial sensors have been discussed. Model selection, calibration equipment, and data selection affect the usefulness of a calibration for a given mission. Calibration is model fitting for a specific mission or class of missions.

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APPENDIX A
CALIBRATION EXAMPLE

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CALIBRATION EXAMPLE

In the following examples, MATLAB¹ is used for all calculations. The calculations are all based on least squares methods or regression analysis. The least squares solution can be developed through a projection argument. Consider the vector equation $Xb=Y$ where the vector b is the unknown. The portion of Y that is contained in the space spanned by X is desired; also, it is possible to project Y into the space spanned by X by forming the inner product of Y with X . However, this operation must be performed on both sides of the equation, which results in $X'Xb=X'Y$. Next, to find b , each side of the equation must be multiplied by the inverse of $X'X$ so $b=inv(X'X)X'Y$. See the references in the text for other and more detailed explanations of least squares. The following series of examples demonstrates the effects of the observation set on parameter estimation. Some metrics for comparing observation sets and for comparing models are introduced.

Example 1

Consider the situation when a straight line is to be fit and ten points are available. To select the ten points, the experimenter may choose each point between one and ten. We may wish to compare this with choosing half the points at zero and half at ten. In order to find the variance of the parameters, we can follow the ensuing procedure. First, the matrix of the independent variables is formed. Then the information matrix $X'X$ is found, and finally, its inverse matrix yields the form of the parameter's covariance. Repeating this process for both observation sets allows a comparison of the observation sets, based on final parameter error.

Assume the range from 1 to 10 is of interest; the first observation set uniformly spaces ten observations across the range of interest. The column of ones is included to estimate the y-intercept, while the second column provides information to estimate the slope of the line.

```
x=[ones(10,1),[1:10]']
1      x =
2      1  1
3      1  2
4      1  3
5      1  4
6      1  5
7      1  6
8      1  7
9      1  8
10     1  9
11     1 10
12
      xtx=x'*x
```

¹ Not an acronym

The following matrix is sometimes called the information matrix. Each entry is the inner product of the corresponding columns. The diagonal elements give the magnitude of each column, while the off-diagonal elements indicate the similarity of the columns.

$$\begin{array}{l}
 \text{xtx} = \\
 \begin{array}{cc}
 10 & 55 \\
 55 & 385
 \end{array}
 \end{array}$$

$$\text{vx} = \text{inv}(\text{xtx})$$

$$\begin{array}{l}
 \text{vx} = \\
 \begin{array}{cc}
 0.4667 & -0.0667 \\
 -0.0667 & 0.0121
 \end{array}
 \end{array}$$

Next, the same procedure is followed to find the form of the covariance associated with an alternate observation set. This alternate set concentrates the observations at the extremes of the range of interest.

$$\text{w} = [1 \ 0; 1 \ 0; 1 \ 0; 1 \ 0; 1 \ 10; 1 \ 10; 1 \ 10; 1 \ 10; 1 \ 10]$$

$$\begin{array}{l}
 \begin{array}{cc}
 1 & \text{w} = \\
 2 & 1 \ 0 \\
 3 & 1 \ 0 \\
 4 & 1 \ 0 \\
 5 & 1 \ 0 \\
 6 & 1 \ 0 \\
 7 & 1 \ 10 \\
 8 & 1 \ 10 \\
 9 & 1 \ 10 \\
 10 & 1 \ 10 \\
 11 & 1 \ 10
 \end{array}
 \end{array}$$

$$\text{wtw} = \text{w}' * \text{w}$$

$$\begin{array}{l}
 \begin{array}{cc}
 50 & \text{wtw} = \\
 51 & 10 \ 50 \\
 52 & 50 \ 500
 \end{array}
 \end{array}$$

$$\text{vw} = \text{inv}(\text{wtw})$$

$$\begin{array}{l}
 \text{vw} = \\
 \begin{array}{cc}
 0.2000 & -0.0200 \\
 -0.0200 & 0.0040
 \end{array}
 \end{array}$$

The diagonal elements of vw are considerably smaller than those for vx, signifying that w is the preferred observation set. The residual variance would be multiplied by the vw matrix for the parameter covariance. The residual variance can only be determined after a fit has been made.

Example 2

Assume that the actual model for a set of data is $.01*x*x+.5*x+.05$. Further assume that the standard deviation of the noise is $.01$. Assume that the model is erroneously assumed to be linear. From the observation sets in Example 1, the observations will be generated and then the estimation of the parameters will be calculated for a linear model.

$$x1=[x,x(:,2).^2]$$

```
x1 =  
1 1 1  
1 2 4  
1 3 9  
1 4 16  
1 5 25  
1 6 36  
1 7 49  
1 8 64  
1 9 81  
1 10 100
```

The x1 matrix contains a column that corresponds to the square of the previous column. Additional columns would represent new independent variables.

$$yx1=x1* [.05; 5; .01]$$

```
yx1 =  
0.5600  
1.0900  
1.6400  
2.2100  
2.8000  
3.4100  
4.0400  
4.6900  
5.3600  
6.0500
```

For the x1-observation set, the vector yx1 gives the true values. Next, noise is added, and finally, the least squares solution provides the parameter vector pxx.

$$yx1noise=.01*randn(size(yx1))$$

```
yx1noise =  
-0.0019  
0.0073  
-0.0059  
0.0218  
-0.0014
```

```
0.0011
0.0107
0.0006
-0.0010
-0.0083
```

```
yx1n=yx1+yx1noise
```

```
yx1n =
0.5581
1.0973
1.6341
2.2318
2.7986
3.4111
4.0507
4.6906
5.3590
6.0417
```

```
pxx=inv(x'*x)*x'*yx1n
```

```
pxx =
-0.1639
0.6093
```

The previous process is repeated for the observation set w.

```
w1=[w,w(:,2).*w(:,2)]
```

```
w1 =
1 0 0
1 0 0
1 0 0
1 0 0
1 0 0
1 10 100
1 10 100
1 10 100
1 10 100
1 10 100
```

```
yw1=w1* [.05;.5;.01]
```

```
yw1 =
0.0500
0.0500
0.0500
0.0500
0.0500
6.0500
6.0500
6.0500
6.0500
```

6.0500
6.0500

Next, the previously generated noise vector is added to the true values

$$yw1n=yw1+yx1noise$$

yw1n =
0.0481
0.0573
0.0441
0.0718
0.0486
6.0511
6.0607
6.0506
6.0490
6.0417

The least squares solution is calculated as follows

$$pw=inv(w'*w)*w'*yw1n$$

pw =
0.0540
0.5997

This parameter set can be compared to the previous set of values. Note that in each of these cases, a linear approximation is being made to a noisy quadratic function. Each observation can be thought of as a vote for information is needed for the relationship. The least squares solution then minimizes the error for a particular functional relationship.

With x1 (which contains a column for the squared parameter), the correct model can be fit as follows.

$$px=inv(x1'*x1)*x1'*yx1n$$

px =
0.0445
0.5051
0.0095

The observation set w cannot be used to fit the three-parameter model. This data set only has two values associated with the independent variable, and three are needed to fit a quadratic term. It can be shown that the ideal observation set for estimation of a quadratic concentrates observations at three points: the extremes of the range of interest and the midpoint.

For the linear fits, the parameters px give a better fit to the quadratic over the range 1-10 than the parameters pw. The parameters pw provide a better fit to the quadratic at the end points of the range and happen to be closer to the actual values. If the range of interest were between 50 and 60, any straight line estimators would be quite different from the lines defined by px and pw. The interplay between the model selected and the observation set chosen is important and affects the quality of the results attained through estimation.

Example 3

To continue the previous examples, residuals will be considered. The residual is the difference between the estimated location and the observed location. An examination of residuals can allow us to select models and predict operating performance.

The next command calculates the residuals for the linear model with equally spaced intervals.

```
xres=yx1n-x*pxx
```

```
xres =  
  0.1127  
  0.0425  
 -0.0299  
 -0.0415  
 -0.0840  
 -0.0808  
 -0.0506  
 -0.0200  
  0.0392  
  0.1125
```

```
xres2=xres'*xres
```

```
xres2 =  
  0.0479
```

The pattern of the residuals from positive to negative and back to positive indicates the model may not be correct. The researcher uses the idea that if the model is correct, then the residuals will be randomly distributed. Many times, a plot of the residuals will reveal information not accounted for by the model. The variance of the residuals is an alternate indicator of how well a model fits the data.

Next, the residuals for the linear model with all the observations at the extremes are calculated.

$$wres = yw1n - w * pw$$

```
wres =  
-0.0059  
0.0033  
-0.0099  
0.0178  
-0.0054  
0.0005  
0.0100  
-0.0000  
-0.0016  
-0.0089
```

$$wres2 = wres' * wres$$

```
wres2 =  
6.7316e-004
```

In this case, the model seems to fit well. The problem is that the observation set only allows for the estimation of intercept and slope; it is not possible to observe a nonlinear component because of the observations selected. A quadratic term is not observable from the observation set. The input or observations preclude its detection.

The residuals for the correct model are now calculated.

$$x3res = yx1n - x1 * px$$

```
x3res =  
-0.0009  
0.0046  
-0.0110  
0.0153  
-0.0083  
-0.0051  
0.0062  
-0.0010  
0.0013  
-0.0012
```

$$x3res2 = x3res' * x3res$$

```
x3res2 =  
5.1387e-004
```

A comparison of this model with the first model can be simply done by examining the residuals and the magnitude of the residuals. The residuals of the quadratic model appear to be randomly distributed and have a smaller magnitude than those of the linear model. Generally, an analysis of residuals will

lead the investigator to accept the model as adequate for the purpose at hand or to pursue a better model. Models are typically suggested by the science associated with the data. The use of empirical methods does not always aid in understanding the phenomena being investigated, although they can be useful for designing interpolative models.

To find the estimate of the parameter variance for a given model, the inverse of the information matrix and the variance of the residuals are needed. The variance of the residuals is properly estimated as the sum of the squares of the residuals divided by the number of observations minus the number of parameters.

Example 4

In this section, the expected variance will be found for a point in the middle of the observation set, a point at the boundary of the observation set, and a point outside the observation set.

First, four points are defined:

$$p1=[1,5.5]$$

$$p1 = \begin{matrix} 1.00000000000000 & 5.50000000000000 \end{matrix}$$

$$p2=[1,1]$$

$$p2 = \begin{matrix} 1 & 1 \end{matrix}$$

$$p3=[1,10]$$

$$p3 = \begin{matrix} 1 & 10 \end{matrix}$$

$$p4=[1,13]$$

$$p4 = \begin{matrix} 1 & 13 \end{matrix}$$

These points are in the middle of the range, on the extremes, and outside the range of observations.

Using the first model, the variance can be estimated as

$$\text{varx}=\text{xres2}/(10-2)$$

$$\text{varx} = 0.00598350305127$$

The covariance of the parameters can be found.

Recall from Example 1 that the form of the covariance matrix was calculated as \mathbf{v}_x .

```
cx=varx*vx
```

```
cx =  
  0.00279230142392 -0.00039890020342  
 -0.00039890020342  0.00007252730971  
p1*cx*p1'
```

```
ans =  
  5.983503051265155e-004  
p2*cx*p2'
```

```
ans =  
  0.00206702832680
```

```
p3*cx*p3'
```

```
ans =  
  0.00206702832680
```

```
p4*cx*p4'
```

```
ans =  
  0.00467801147644
```

It is not a coincidence that the points that equal distance from the center of observations have the same expected error.

For the second model, the previous process is repeated. First, the covariance matrix of the parameters is found.

```
cw=(wres2/8)*vw
```

```
cw =  
  1.0e-004 *  
  0.16828959106776 -0.01682895910678  
 -0.01682895910678  0.00336579182136
```

The following command calculates all four variances as a row vector.

```
[p1*cw*p1' p2*cw*p2' p3*cw*p3' p4*cw*p4']
```

```
ans =  
  1.0e-004 *  
  0.08498624348922  0.13799746467556  0.16828959106776  0.29955547210062
```

Note that in this, the center of the observation data is at five, rather than at 5.5; thus, p2 and p3 are not equal distance from the center. For the three-parameter model, the following calculations define the expected error.

$$cx1 = x3res2 / 7 * inv(x1' * x1)$$

$$cx1 =$$

$$1.0e-003 *$$

0.10155049138894	-0.03854024673195	0.00305874974063
-0.03854024673195	0.01771294167983	-0.00152937487032
0.00305874974063	-0.00152937487032	0.00013903407912

Next, the points have to be in the parameter space, so a squared term needs to be included in each point.

$$p1 = [p1 \ 5.5 * 5.5]; p2 = [p2 \ 1]; p3 = [p3 \ 100]; p4 = [p4 \ p4(2) * p4(2)];$$

$$[p1 * cx1 * p1' \ p2 * cx1 * p2' \ p3 * cx1 * p3' \ p4 * cx1 * p4']$$

$$ans =$$

$$1.0e-003 *$$

0.01680400638759	0.04538072342463	0.04538072342463	0.37772778615207
------------------	------------------	------------------	------------------

Notice that the variance associated with p2 and p3 is equal.

The two most common methods of comparing models are based on R-squared and the adjusted R-squared values. These values compare the variation explained by the model to the total variation in the observations. The total variation is simply the square of the differences between each observation and the average of the observations. The variation explained by the model can be ascertained by removing the residual sum of squares from the total variation. The R-squared value is the ratio of the variation explained by the model to the total variation and will be between zero and one. Each model will have an R-squared value and these can be used for comparison of models. If the models being compared have the same number of parameters, R-squared is used, and if the number of parameters for each model differs, then the adjusted R-squared is used. To find the total variation in the observations, the variance of the observations can be multiplied by N-1 where N is the number of observations.

To find the R-squared value for the first model,

$$tssyx = 9 * var(yx1n)$$

$$tssyx =$$

$$30.6765$$

The following formula is equivalent to the variation explained by the model and divided by the total amount of variation in the data.

```

R2m1=1-xres2/tssyx
R2m1 =
  0.9984
tssyw=9*var(yw1n)
tssyw =
  89.8996
format long
R2w=1-wres2/tssyw
R2w =
  0.99999251210673

```

Based on the comparison of R-squared, the second model seems to be better.

To compare models with different numbers of parameters, the adjusted R-squared value for each model needs to be calculated. The adjustment is to divide each sum of squares by the number of observations diminished by the number of parameters used for the model. For the total sum of squares, the adjustment consists of dividing by N-1 (since the mean is used to find this value). The residual sum of squares is divided by N-k where k is the number of parameters in the model. The calculations are similar to the previous case.

For the first model,

```

R2am1=1-9*xres2/tssyx/7
R2am1 =
  0.99799375308575

```

For the second model,

```

R2aw=1-wres2/tssyw*9/7
R2aw =
  0.99999037270866

```

For the third (and true) model,

```

R2am3=1-9*x3res2/tssyx/6
R2am3 =
  0.99997487310096

```

A comparison of these values shows the true model does not receive the best adjusted R-square value. The problem is in the selection of the observation sets. Recall that if it were known because of a *priori* knowledge that the model was quadratic, the ideal observation set would be to equally divide the observations between the two extremes and the midpoint.

Statistical packages aid the researcher in this type of analysis by simplifying calculations and providing safeguards for the estimation process. This series of examples illustrates the desirability of a well-thought-out observation set. It should be clear that a model will most accurately represent the dependent variable in the region of the observations.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE October 2000	3. REPORT TYPE AND DATES COVERED Final
4. TITLE AND SUBTITLE Calibration of Inertial Sensors			5. FUNDING NUMBERS PR: 1L162618AH80
6. AUTHOR(S) Thompson, A.A. (ARL)			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Research Laboratory Weapons & Materials Research Directorate Aberdeen Proving Ground, MD 21005-5066			8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Laboratory Weapons & Materials Research Directorate Aberdeen Proving Ground, MD 21005-5066			10. SPONSORING/MONITORING AGENCY REPORT NUMBER ARL-MR-481
11. SUPPLEMENTARY NOTES			
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE
13. ABSTRACT (Maximum 200 words) The calibration of accelerometers, angular rate sensors, and inertial measurement units (IMU) increases the accuracy of their measurements. For artillery rounds and most rockets, measurements are acquired while the body is spinning. By the comparison of the sensor output to known input, calibration is accomplished. Calibration requires a model of the sensor. The model represents the measurement process. Model selection, calibration equipment, and data selection affect the usefulness of a calibration for a given mission. Calibration is model fitting for a specific mission or class of missions.			
14. SUBJECT TERMS calibration inertial sensors			15. NUMBER OF PAGES 30
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT