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# A Conformal-Mapping Treatment of the Effect of a Semi-Infinite Gate on a Two-Dimensional Electron Gas

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Sensors and Electron Devices Directorate

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## Abstract

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A pinched-off high-electron-mobility transistor containing a perfectly conducting two-dimensional electron gas (2DEG) is described mathematically within an idealized two-dimensional geometry, so that conformal mapping techniques can be used to compute internal fields at the transistor drain. The field and charge distribution at the drain end of the 2DEG calculated in this way suggest that the charge is a nonmonotonic function of position in this region.

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# Contents

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|          |  |           |
|----------|--|-----------|
| <b>1</b> | <b>Introduction</b>                        | <b>1</b>  |
| <b>2</b> | <b>Schwartz-Christoffel Transformation</b> | <b>3</b>  |
| <b>3</b> | <b>Perfectly Conducting 2DEG</b>           | <b>8</b>  |
| <b>4</b> | <b>Conclusions</b>                         | <b>10</b> |
|          | <b>References</b>                          | <b>11</b> |
|          | <b>Distribution</b>                        | <b>13</b> |
|          | <b>Report Documentation Page</b>           | <b>15</b> |

## Figures

|   |   |   |
|---|---|---|
| 1 | Cross section of FET geometry on drain side of 2DEF FET . . . . .                                       | 2 |
| 2 | Physical ( $z$ ) plane for Schwartz-Christoffel mapping . . . . .                                       | 2 |
| 3 | Schwartz-Christoffel plane . . . . .  | 3 |
| 4 | Branch cuts for $w(z)$ . . . . .  | 4 |
| 5 | Graphical solution to equation for $c$ , which locates mapping point $q$ that terminates 2DEG . . . . . | 6 |
| 6 | Density distribution of perfectly conducting 2DEG . . . . .   | 9 |

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# 1. Introduction

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The physics of field-effect transistors (FETs) fundamentally differs from that of bipolar transistors and heterojunction-bipolar transistors (HBTs), in which currents flow perpendicular to layers of various materials. This difference arises from the lateral geometry that underlies the operation of FETs, i.e., the modulation of a “horizontal” current that arises from a drain-to-source voltage by “vertical” electric fields generated by a gate electrode. The FET geometry gives rise to internal electric fields that are complicated and intrinsically two-dimensional (2D), so that the device terminal characteristics (source, drain, and gate voltages and currents) become difficult to predict and dependent on many internal parameters. The geometry makes the physics of 2D electron-gas (2DEG) FETs (high-electron-mobility transistors—HEMTs) especially difficult to unravel, since the quantum-mechanical nature of the electron dynamics in the 2D channel cannot easily be disentangled from the electrostatic problem of finding the fields in the neighboring dielectric materials.

A problem of particular interest to device designers is to determine what happens to the 2DEG as it emerges from under the gate region. Because the electric fields are no longer screened by the gate, the capacitance of the electron gas per unit length with respect to the gate must change, since field lines can now escape to infinity. This problem is made even more interesting by the existence of plasma oscillations predicted by Dyakonov and Shur in 1993 [1], which are profoundly different for a gated and an ungated electron gas [2]. In this report, conformal mapping is used to compute the change in field configuration of the electron gas in this situation, with the extreme assumption of a perfectly conducting electron gas. The finite-conductivity effects needed for the study of plasma oscillations will be the subject of a subsequent report.

Figure 1 is a cross section of the device geometry near the drain. The 2DEG is assumed to be a vertical distance  $h$  below the gate and to terminate at a horizontal distance  $q$  beyond the drain end of the gate. Let us begin with the symmetrized version of this geometry shown in figure 2 (resembling that of a junction FET (JFET) [3]) and tailored for a treatment based on complex-variable theory. The new geometry, which is embedded in the complex plane  $z = x + iy$ , has the following idealized features:

1. There are two gates separated by a distance  $h$ , with the 2DEG between them along the  $x = \text{Re } z$  axis and ending at a point  $x = q > 0$  on the real axis.
2. The 2DEG is infinitely extended to the left.
3. The gates are both assumed to be infinitely thick, with vertical boundaries along the  $y = \text{Im } z$ -axis.

4. All dielectric constants are set equal to one.

This idealized geometry will allow us to set up the conformal-mapping problem discussed in the next section.

Figure 1. Cross section of FET geometry on drain side of 2DEF FET: (a) real and (b) idealized.

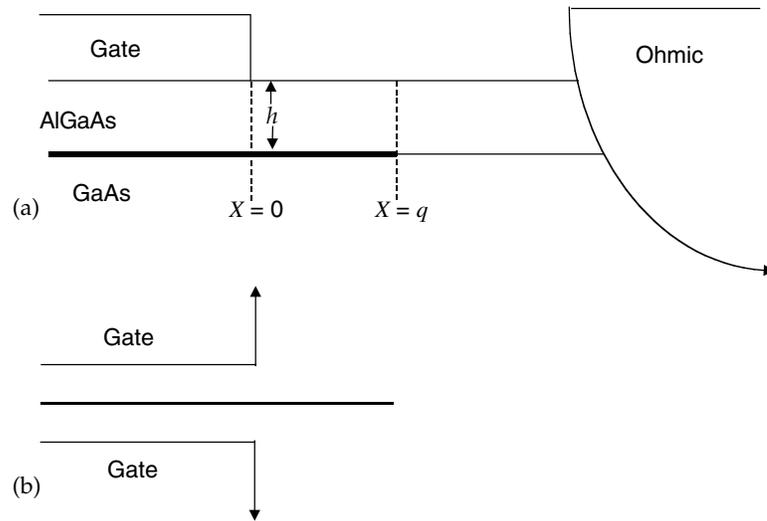
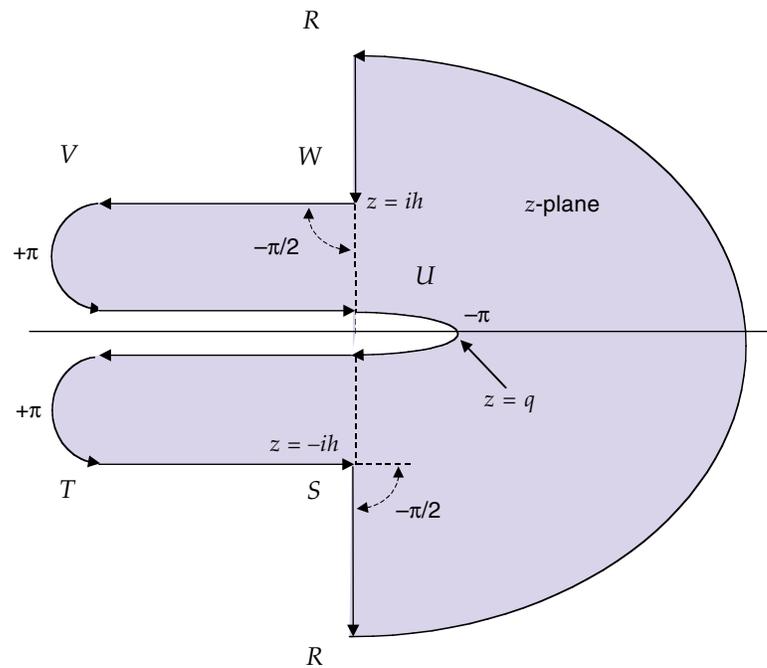


Figure 2. Physical ( $z$ ) plane for Schwartz-Christoffel mapping.



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## 2. Schwartz-Christoffel Transformation

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Following standard procedures [4], I seek a complex-variable function that will map the shaded area in figure 2 (the  $z$ -plane) into the upper half of a new complex-variable plane, the  $w$ -plane. The special points  $R, S, T, U, V,$  and  $W$  identify the right-hand half-space ( $R$ ), the lower gate edge ( $S$ ), points deep inside the gated channel and below the 2DEG ( $T$ ), the end of the 2DEG ( $U$ ), points deep inside the gated channel and above the 2DEG ( $V$ ), and the upper gate edge ( $W$ ). The Schwartz-Christoffel recipe treats this geometry as a seven-sided polygon with the point  $R$  at infinity. Because of the symmetry, we can assume that the points in the complex  $z$ -plane other than  $R$  pair symmetrically in the  $w$ -plane; this results in the following differential equation for the Schwartz-Christoffel mapping:

$$\frac{dz}{dw} = A \frac{w\sqrt{w^2 - c^2}}{w^2 - 1}, \quad (1)$$

where  $A$  and  $c$  are to be determined. The function  $w(z)$  maps the points  $R, S, T, U, V,$  and  $W$  in the  $z$ -plane into the points in figure 3 on the real axis of the complex  $w$ -plane. The problem now reduces to

1. solving this equation,
2. finding values of  $A$  and  $c$ ,
3. solving the Laplace equation in the  $w$ -plane, and
4. inverting the transformation to return to the  $z$ -plane.

Figure 4 shows the proper definition of the branch cuts for the function  $\sqrt{w^2 - c^2} : \arg(w - c) \in [-\pi, \pi], \arg(w + c) \in [-\pi, \pi]$ .

Figure 3.  
Schwartz-Christoffel  
plane.

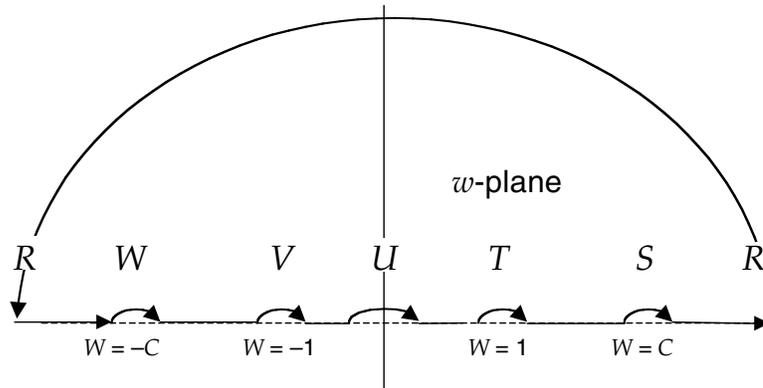
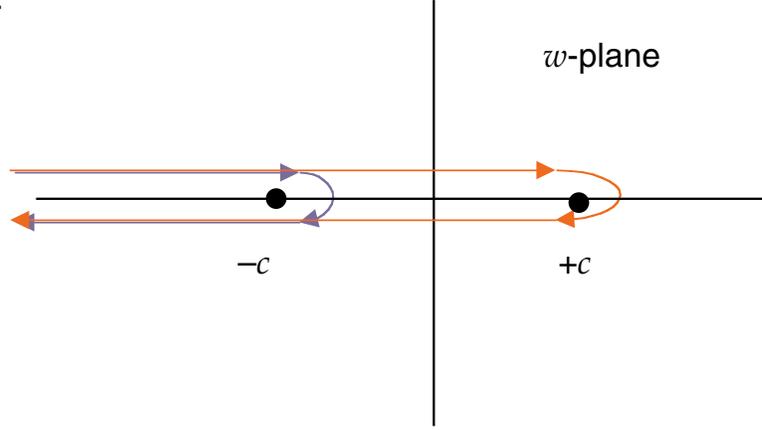


Figure 4. Branch cuts for  $w(z)$ .



The differential equation is trivially solved for the function  $z(w)$ :

$$z(w) = z_0 + A \int_c^w \frac{\sqrt{\xi^2 - c^2}}{\xi^2 - 1} \xi d\xi = z_0 + A \left[ \sqrt{\frac{w^2 - c^2}{c^2 - 1}} - \tan^{-1} \sqrt{\frac{w^2 - c^2}{c^2 - 1}} \right], \quad (2)$$

which we can rewrite as

$$z(w) = z_0 + A \left[ \sqrt{\frac{w^2 - c^2}{c^2 - 1}} - \frac{1}{2i} \ln \left\{ \frac{1 + i\sqrt{\frac{w^2 - c^2}{c^2 - 1}}}{1 - i\sqrt{\frac{w^2 - c^2}{c^2 - 1}}} \right\} \right]. \quad (3)$$

Now, consider the action of this mapping from various regions of  $\text{Re } w$  to the  $z$ -plane:

1.  $w \in [-\infty, -c]$  (segment  $RW$ ): for large  $w$ ,  $z \approx z_0 + A \frac{w}{\sqrt{c^2 - 1}} \rightarrow +i\infty$ , so that  $A = iB$  with  $B < 0$ . As  $w \rightarrow c$ , the term in brackets vanishes; since  $w = c$  must correspond to  $z = +ih$  in the physical  $z$ -plane, we have  $z_0 = ih$ . Then

$$z = ih + iB \left[ \sqrt{\frac{w^2 - c^2}{c^2 - 1}} - \tan^{-1} \sqrt{\frac{w^2 - c^2}{c^2 - 1}} \right]. \quad (4)$$

2.  $w \in [-c, -1]$  (segment  $WV$ ): On this segment of  $\text{Re } w$ , we have  $\sqrt{\frac{w^2 - c^2}{c^2 - 1}} = i\sqrt{\frac{c^2 - w^2}{c^2 - 1}}$ , with  $\sqrt{\frac{c^2 - w^2}{c^2 - 1}} < 1$ , according to our choice of branch cut, so we can rewrite  $z(w)$  as

$$\begin{aligned} z &= ih + iB \left[ i\sqrt{\frac{c^2 - w^2}{c^2 - 1}} - \frac{1}{2i} \ln \left\{ \frac{1 - \sqrt{\frac{c^2 - w^2}{c^2 - 1}}}{1 + \sqrt{\frac{c^2 - w^2}{c^2 - 1}}} \right\} \right] \\ &= ih - B \left[ \sqrt{\frac{c^2 - w^2}{c^2 - 1}} - \frac{1}{2} \ln \left\{ \frac{1 + \sqrt{\frac{c^2 - w^2}{c^2 - 1}}}{1 - \sqrt{\frac{c^2 - w^2}{c^2 - 1}}} \right\} \right] \\ &= ih - B \left[ \sqrt{\frac{c^2 - w^2}{c^2 - 1}} - \tanh^{-1} \sqrt{\frac{c^2 - w^2}{c^2 - 1}} \right]. \end{aligned} \quad (5)$$

From this we see that  $w \rightarrow -1 \Rightarrow z \rightarrow ih - \infty$ , so long as  $B < 0$  and real. This takes care of segment  $WV$ .

3.  $w \in [-1, 0]$  (segment  $VU$ ): On this segment of  $\text{Re } w$ , we have  $\sqrt{\frac{c^2-w^2}{c^2-1}} > 1$ , so that the hyperbolic tangent is affected. Write

$$\frac{1}{2} \ln \left\{ \frac{1 + \sqrt{\frac{c^2-w^2}{c^2-1}}}{1 - \sqrt{\frac{c^2-w^2}{c^2-1}}} \right\} = \frac{1}{2} \ln \left\{ \frac{\sqrt{\frac{c^2-w^2}{c^2-1}} + 1}{(-1) \left( \sqrt{\frac{c^2-w^2}{c^2-1}} - 1 \right)} \right\} \quad (6)$$

and pick the log branch cut such that  $\ln(-x) = -i\pi + \ln(x)$ . Then

$$z = \frac{1}{2} \ln \left\{ \frac{1 + \sqrt{\frac{c^2-w^2}{c^2-1}}}{1 - \sqrt{\frac{c^2-w^2}{c^2-1}}} \right\} + \frac{i\pi}{2} = \tanh^{-1} \left( \sqrt{\frac{c^2-1}{c^2-w^2}} \right) + \frac{i\pi}{2} \quad (7)$$

and so

$$z = ih - B \left[ \sqrt{\frac{c^2-w^2}{c^2-1}} - \tanh^{-1} \left( \sqrt{\frac{c^2-1}{c^2-w^2}} \right) - \frac{i\pi}{2} \right]. \quad (8)$$

Now, let us choose the point  $z = q$ , where  $q$  is real, to correspond to  $w = 0$ . This will be true if  $B = -\frac{2h}{\pi}$  and

$$q = \frac{2h}{\pi} \left[ \frac{c}{\sqrt{c^2-1}} - \tanh^{-1} \left( \sqrt{\frac{c^2-1}{c}} \right) \right]. \quad (9)$$

This equation now determines  $c$  in terms of the real parameter  $q$ . Then

$$z = \frac{2h}{\pi} \left[ \sqrt{\frac{c^2-w^2}{c^2-1}} - \tanh^{-1} \left( \sqrt{\frac{c^2-1}{c^2-w^2}} \right) \right] \quad (10)$$

and  $w \rightarrow -1 \Rightarrow z \rightarrow -\infty$  (from the inverse hyperbolic tangent), while  $w \rightarrow 0 \Rightarrow z \rightarrow q$ .

Let  $x = \frac{\sqrt{c^2-1}}{c}$ . Then equation (9) becomes

$$\frac{1}{x} - \frac{\pi q}{2h} = \tanh^{-1}(x). \quad (11)$$

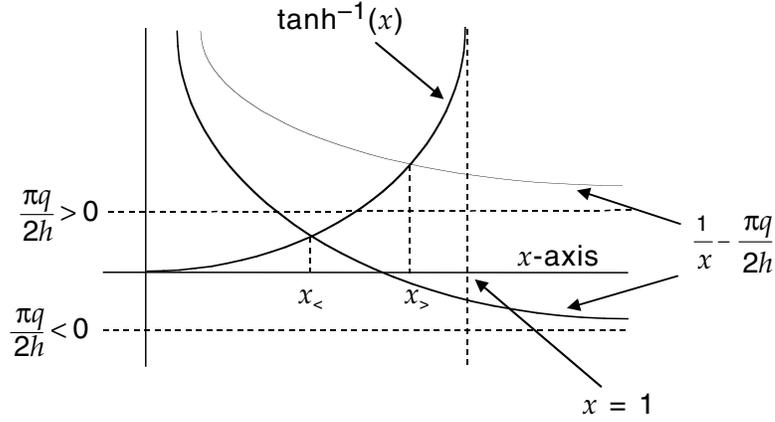
The plot in figure 5 shows that there is an acceptable root  $x$  for every  $q \in [-\infty, \infty]$ , where the root  $x$  is always in the range  $[0, 1]$ .

4.  $w \in [0, 1]$  (segment  $UT$ ): This is the same as  $w \in [-1, 0]$ , since the function depends on  $w^2$ , but it runs backwards, and  $w \rightarrow 1 \Rightarrow z \rightarrow -\infty$ ,  $w \rightarrow 0 \Rightarrow z \rightarrow q$  once more.

5.  $w \in [1, c]$  (segment  $TS$ ): Here we have

$$z = \frac{2h}{\pi} \left[ \sqrt{\frac{c^2-w^2}{c^2-1}} - \tanh^{-1} \left( \sqrt{\frac{c^2-1}{c^2-w^2}} \right) \right]. \quad (12)$$

Figure 5. Graphical solution to equation for  $c$ , which locates mapping point  $q$  that terminates 2DEG;  $x_>$  is root for  $\pi q/2h > 0$ , and  $x_<$  is root for  $\pi q/2h < 0$ .



Since  $w > 1 \Rightarrow \sqrt{\frac{c^2-1}{c^2-w^2}} > 1$ , we have

$$\begin{aligned}
 -\tanh^{-1}\left(\sqrt{\frac{c^2-1}{c^2-w^2}}\right) &= -\frac{1}{2} \ln \left\{ \frac{1 + \sqrt{\frac{c^2-1}{c^2-w^2}}}{1 - \sqrt{\frac{c^2-1}{c^2-w^2}}} \right\} \\
 &= \frac{1}{2} \ln \left\{ \frac{(-1) \left( \sqrt{\frac{c^2-1}{c^2-w^2}} - 1 \right)}{\sqrt{\frac{c^2-1}{c^2-w^2}} + 1} \right\} \\
 &= \frac{1}{2} \ln \left\{ \frac{1 - \sqrt{\frac{c^2-w^2}{c^2-1}}}{1 + \sqrt{\frac{c^2-w^2}{c^2-1}}} \right\} - \frac{i\pi}{2} \\
 &= -\tanh^{-1}\left(\sqrt{\frac{c^2-w^2}{c^2-1}}\right) - \frac{i\pi}{2}. \quad (13)
 \end{aligned}$$

Then

$$\begin{aligned}
 z &= \frac{2h}{\pi} \left[ \sqrt{\frac{c^2-w^2}{c^2-1}} - \tanh^{-1}\left(\sqrt{\frac{c^2-w^2}{c^2-1}}\right) - \frac{i\pi}{2} \right] \\
 &= -ih + \frac{2h}{\pi} \left[ \sqrt{\frac{c^2-w^2}{c^2-1}} - \tanh^{-1}\left(\sqrt{\frac{c^2-w^2}{c^2-1}}\right) \right] \quad (14)
 \end{aligned}$$

on this segment. Note that  $w \rightarrow 1 \Rightarrow z \rightarrow -ih - \infty$ , as it should.

6.  $w \in [c, \infty]$  (segment  $SR$ ): On this segment we have  $\sqrt{\frac{c^2-w^2}{c^2-1}} = -i\sqrt{\frac{w^2-c^2}{c^2-1}}$ , so that

$$\begin{aligned}
 \tanh^{-1}\left(-i\sqrt{\frac{w^2-c^2}{c^2-1}}\right) &= \frac{1}{2} \ln \left( \frac{1 - i\sqrt{\frac{w^2-c^2}{c^2-1}}}{1 + i\sqrt{\frac{w^2-c^2}{c^2-1}}} \right) \\
 &= -\frac{1}{2} \ln \left( \frac{1 + i\sqrt{\frac{w^2-c^2}{c^2-1}}}{1 - i\sqrt{\frac{w^2-c^2}{c^2-1}}} \right)
 \end{aligned}$$

$$\begin{aligned}
&= -i \frac{1}{2i} \ln \left( \frac{1 + i\sqrt{\frac{w^2 - c^2}{c^2 - 1}}}{1 - i\sqrt{\frac{w^2 - c^2}{c^2 - 1}}} \right) \\
&= -i \tan^{-1} \left( \sqrt{\frac{w^2 - c^2}{c^2 - 1}} \right) \quad (15)
\end{aligned}$$

and

$$\begin{aligned}
z &= -ih + \frac{2h}{\pi} \left[ -i\sqrt{\frac{w^2 - c^2}{c^2 - 1}} + i \tan^{-1} \left( \sqrt{\frac{w^2 - c^2}{c^2 - 1}} \right) \right] \\
&= -ih - \frac{2ih}{\pi} \left[ \sqrt{\frac{w^2 - c^2}{c^2 - 1}} - \tan^{-1} \left( \sqrt{\frac{w^2 - c^2}{c^2 - 1}} \right) \right]. \quad (16)
\end{aligned}$$

Clearly, for large real  $w > 0$ , we have  $z \approx -ih - \frac{2ih}{\pi} \frac{w}{\sqrt{c^2 - 1}} \rightarrow -i\infty$ , which is physically correct. Note that the Schwartz reflection principle implies that if  $w = u + iv$  and  $z$  is a function of  $w^2$ , then

$$z(\{-u + iv\}^2) = z(\{w^*\}^2) = z(\{w^2\}^*) = z^*(w^2). \quad (17)$$

Hence, moving to  $\text{Re } w < 0$  in the upper half of the  $w$ -plane takes you to the lower half of the  $z$ -plane.

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### 3. Perfectly Conducting 2DEG

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Here  $V = 0$  on the electrodes, and  $V = V_0$  on the 2DEG between them. Then the complex potential is

$$V(z) = \frac{V_0}{\pi} \operatorname{Im} \left\{ \ln \left( \frac{w(z) - 1}{w(z) + 1} \right) \right\} \equiv \operatorname{Im} U(w(z)), \quad (18)$$

which reproduces the expression given in Churchill and Brown [5]. The electric field components are

$$E_x = \frac{\partial V}{\partial x} = \operatorname{Im} \left[ \frac{\partial U}{\partial x} \right] = \operatorname{Im} \left[ \frac{\partial U}{\partial z} \right] = \operatorname{Im} [U'(w)w'(z)] \quad (19)$$

and

$$E_y = \frac{\partial V}{\partial y} = \operatorname{Im} \left[ \frac{\partial U}{\partial y} \right] = \operatorname{Im} \left[ i \frac{\partial U}{\partial z} \right] = \operatorname{Re} [U'(w)w'(z)]. \quad (20)$$

But

$$\frac{dw}{dz} = \left\{ \frac{dz}{dw} \right\}^{-1} = \frac{i\pi}{2h} \frac{w^2 - 1}{w\sqrt{w^2 - c^2}} \quad (21)$$

and

$$\frac{\partial U}{\partial z} = \frac{2V_0}{\pi} \frac{1}{w^2 - 1}, \quad (22)$$

so that

$$\begin{aligned} U'(w)w'(z) &= \frac{2V_0}{\pi} \frac{1}{w^2 - 1} \frac{i\pi}{2h} \frac{w^2 - 1}{w\sqrt{w^2 - c^2}} \\ &= \frac{iV_0}{h} \frac{1}{w\sqrt{w^2 - c^2}}. \end{aligned} \quad (23)$$

Since  $|w| < c$  everywhere on the  $z$ -axis, we can write

$$\begin{aligned} E_x &= \frac{V_0}{h} \operatorname{Im} \frac{1}{w\sqrt{c^2 - w^2}}, \\ E_y &= \frac{V_0}{h} \operatorname{Re} \frac{1}{w\sqrt{c^2 - w^2}}, \quad \text{and} \\ |E| &= \frac{V_0}{h} \left| \frac{1}{w\sqrt{c^2 - w^2}} \right|. \end{aligned} \quad (24)$$

A few things to note:

1.  $E_x = 0$  for real  $w$ ; i.e., the vector  $E$  is normal to the  $\operatorname{Re} z$  axis, as it should be for a perfect conductor.
2. The field has a  $|z|^{-1/2}$  singularity at the end of the 2DEG.
3. The field has  $|c \pm z|^{-1/3}$  singularities at the electrode corners.

4. At large  $z$ ,  $\text{Re } z > 0$  (or large  $w$ ,  $\text{Re } w > 0$ ), the field goes as  $|z|^{-2}$ , i.e., like a dipole. Hence, the electrode charges completely screen out the 2DEG charge at large distances.
5. For large  $z$ ,  $\text{Re } z < 0$ , and  $\text{Im } z[-ih, ih]$  (or  $w \rightarrow \pm 1$ ), the field approaches a constant:

$$E_y = \pm \frac{V_0}{h} \frac{1}{\sqrt{c(q)^2 - 1}}, \quad (25)$$

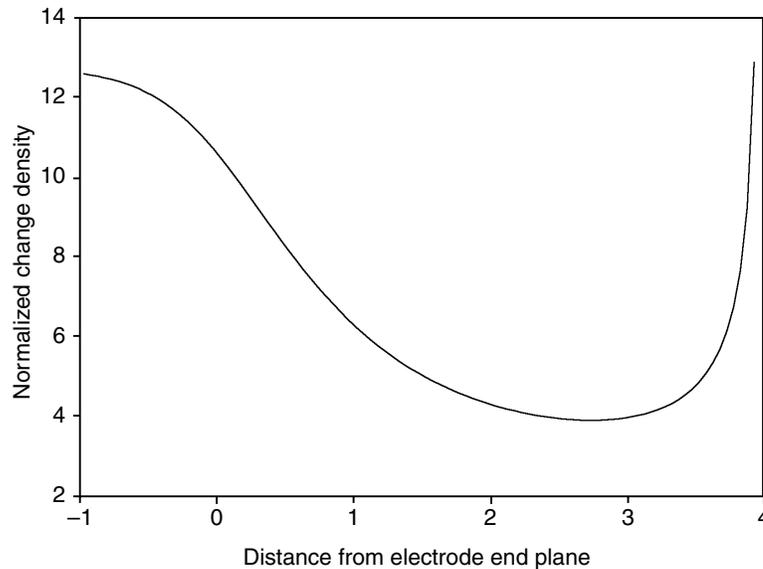
where the sign is the same as that of  $\text{Im } z$ . This is also correct, since the problem is  $x$ -independent in this limit. Since  $C = \frac{Q}{V_0}$  and  $Q = 2E_y$  by Gauss's law, we obtain the following nontrivial fringing capacitance/length,

$$C = \frac{2\epsilon}{h} \frac{1}{\sqrt{c(q)^2 - 1}}, \quad (26)$$

associated with the termination of the gate.

Figure 6 shows a numerical calculation of the 2D density of a perfectly conducting 2DEG using the expressions derived in this report and the parameter values  $q = 4$ ,  $h = 1$  (only their ratio enters in, so the units are arbitrary). Note that in a plasma-active device structure, the potential and density would be coupled in a complicated way, with both acting as degrees of freedom, rather than having the potential as a constant. Despite the oversimplification, however, the variation of the 2DEG density is highly nontrivial, with a minimum about 80 percent of the way to the end of the gate. This is presumably because the image fields from the sharp corners are trying to hold the electrons within the region between the electrodes, which competes with their natural tendency to accumulate at the end of the gate. The fringing capacitance here is about  $12.6 \epsilon/h$  for this geometry.

Figure 6. Density distribution of perfectly conducting 2DEG. Note divergence at endpoint ( $z = 4$ ).



---

## 4. Conclusions

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The predicted form of the charge density near the end of the gate suggests that plasma oscillations that propagate to this point will see a depletion region. This corresponds to a region of decreased Dyakonov-Shur plasma velocity, which in turn suggests that plasma waves will see an impedance mismatch between the gated and ungated regions over and above what would be expected from their intrinsic dissimilarity. Because the central assumption of the Dyakonov-Shur plasma model of a 2DEG FET is a special set of boundary conditions at the gate and source, the existence of this “natural” boundary condition may require that their assumptions be reexamined, and could perhaps explain why plasma oscillations have yet to be observed in standard HEMTs.

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