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Efficient Solution of the Long-Rod Penetration Equations of Alekseevskii-Tate

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1. Background

The penetration equations that describe the behavior of a long rod that erodes while it penetrates at high velocity were formulated independently by Alekseevskii [1] and Tate [2] in the mid-1960s and are given by

$$L\dot{V} = -Y/\rho_R \quad \text{(rod deceleration)}, \tag{1}$$

$$\frac{1}{2}\rho_R(V-U)^2 + Y = \frac{1}{2}\rho_T U^2 + R \quad \text{(interface stress balance)}, \tag{2}$$

$$V = U - \dot{L} \quad \text{(erosion kinematics), and} \tag{3}$$

$$\dot{P} = U \quad \text{(penetration definition).} \tag{4}$$

In these equations, V is the rod velocity, U is the penetration rate, P is the rod penetration, and L is the rod length, all functions of time. The constant parameters include the rod strength Y, the target resistance R, and the target-to-rod density ratio $\gamma = \rho_T/\rho_R$. The dots signify time differentiation. These equations have typically been integrated numerically to achieve a solution. A decade ago, Walters and Segletes [3] obtained an exact solution to these equations. However, the solution was not expressed in terms of the primitive variables that appear in the original equations, but rather in terms of an oblique transformation variable that was presented without explanation. Furthermore, little attempt was made to collate variables into an orderly fashion, thus leaving an incomplete sense for the term groupings that actually drive the solution. While mathematically rigorous, the solution was somewhat cumbersome to use.

This equation set has been re-examined, in search of improvements and extensions to the solution method. Several improvements to the solution approach are offered herein to improve the solution efficiency. A primary hindrance of the original solution was in the evaluation of the rod velocity as a function of time. While this hindrance remains with the current approach, it may be circumvented by choosing an independent variable other than time, in the evaluation of rod erosion. Indeed, it is often more useful to express the solution in terms of, for example, rod velocity, rather than the canonical function-of-time solution. And while this alternative was available to the original solution [3], the presentation of the original solution perhaps left the false impression with the reader that the numerical evaluation of $V(t)$ was a necessary intermediate step in the solution of $L(V)$ and $P(V)$.

Though the governing equations (1)–(4) pertain only to the time during which penetration and erosion simultaneously occur, extensions to the original solution [3] are herein provided for the subsequent stage of rigid-body penetration or rigid-target rod erosion. In addition to the general-case solution to the penetration problem being addressed, several special-case conditions, including the cases for which $R = Y$ and $\rho_R = \rho_T$, respectively, will also be solved. Not addressed herein, however, because of their simplicity, are three special cases for which $R = 0$, $Y = 0$, and $R = Y = 0$, respectively. The present method, described subsequently, can be used to describe the $R = 0$ solution up until the moment that rigid-body penetration commences. Subsequent behavior, however, will be governed by Poncelet flow. In the case of both $Y = 0$ and $R = Y = 0$, the solution becomes trivial in that the rod velocity remains constant until the rod is totally consumed, at which point the event ceases. The penetration velocity and rod erosion rate also remain constant for these cases, in accordance with equations (2) and (3). For the case of $R = Y = 0$, the steady-state erosion rates are governed by the Bernoulli equation.

2. Closed Form Solution for *L*(*V*)

Without delay, we present the solution to the rod erosion equations, which is valid for all cases (special cases [$R = Y$, $\rho_R = \rho_T$] and the general case):

$$\frac{L}{L_0} = \left(\frac{\sqrt{\gamma} U - \dot{L}}{\sqrt{\gamma} U_0 - \dot{L}_0} \right)^{\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y} - 1\right)} \exp\left[\frac{V_0 \dot{L}_0 - V \dot{L}}{2Y/\rho_R} \right], \tag{5}$$

where the "0" subscripts signify conditions at the onset of the penetration event. It is worthy to note that while $-\dot{L}$ is the rate of rod erosion, the term $\sqrt{\gamma} U$ would be the rate of rod erosion were the case hydrodynamic (i.e., where R=Y=0).

The presentation of the solution, given by equation (5), is obtained by solving equation (1) for L and differentiating with respect to time; then, using equations (2) and (3) to obtain dL/dt in terms of rod velocity V; and finally eliminating dL/dt from the two resulting equations, which gives d^2V/dt^2 in terms of dV/dt and V. The particular expression for dL/dt varies depending on whether the special or general problem cases are considered, and this will affect the form of the governing equation, as will be shown. The resulting equation is integrated to provide dV/dt in terms of V. While the traditional technique is to separate variables and attempt to integrate again to obtain $V(t)$, as was done by Walters and Segletes [3], this step is not necessary to obtain $L(V)$. Equation (1) provides a direct algebraic link between L and dV/dt, and thereby allows dV/dt to be eliminated in favor of L, immediately following the first integration. The result is $L(V)$, which is a desirable way to express the result, as an alternative to $L(t)$.

When special- and general-case problems are considered, the solutions, at first glance, take on different appearances. However, equation (5) was discerned from those solutions (given here, expressed in terms of a single independent variable, V, the rod velocity) by realizing that the various grouping of V terms in the various $L(V)$ solutions all satisfy the elegant form of equation (5):

$R = Y$:

$$\text{Governing Equation:} \quad \frac{\sqrt{\gamma}}{1+\sqrt{\gamma}} V\dot{V}^2 = -\frac{Y}{\rho_R}\ddot{V}, \tag{6}$$

$$\frac{L}{L_0} = \exp\left[\frac{-\rho_R\sqrt{\gamma}}{2Y(1+\sqrt{\gamma})}\left(V_0^2 - V^2\right)\right], \tag{7}$$

$\gamma = 1$:

$$\text{Governing Equation:} \quad \left(\frac{V}{2} + \frac{R-Y}{\rho_R V}\right)\dot{V}^2 = -\frac{Y}{\rho_R}\ddot{V}, \tag{8}$$

$$\frac{L}{L_0} = \left(\frac{V}{V_0}\right)^{\left(\frac{R}{Y}-1\right)} \exp\left[\frac{-\rho_R}{4Y}\left(V_0^2 - V^2\right)\right], \tag{9}$$

General case:

$$\text{Governing Equation:} \quad \frac{1}{1-\gamma}\left(-\gamma V + \sqrt{\gamma V^2 + 2(R-Y)(1-\gamma)/\rho_R}\right)\dot{V}^2 = -\frac{Y}{\rho_R}\ddot{V}, \tag{10}$$

$$\frac{L}{L_0} = \left(\frac{V}{V_0}\cdot\frac{1+\sqrt{1+2(R-Y)(1-\gamma)/(\gamma\rho_R V^2)}}{1+\sqrt{1+2(R-Y)(1-\gamma)/(\gamma\rho_R V_0^2)}}\right)^{\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \cdot \exp\left[\frac{-\rho_R\sqrt{\gamma}}{2Y(1+\sqrt{\gamma})} \times \right.$$
$$\left.\left(V_0^2\cdot\frac{\sqrt{1+2(R-Y)(1-\gamma)/(\gamma\rho_R V_0^2)}-\sqrt{\gamma}}{1-\sqrt{\gamma}} - V^2\cdot\frac{\sqrt{1+2(R-Y)(1-\gamma)/(\gamma\rho_R V^2)}-\sqrt{\gamma}}{1-\sqrt{\gamma}}\right)\right]. \tag{11}$$

Equations (7), (9), and (11) have been organized and presented in a manner to demonstrate the functional linkage between the special- and general-case solutions. For example, when either $R = Y$ or $\gamma = 1$, the extended square-root terms of equation (11) become unity, leading to the simpler (V/V_0) monomial and $(V_0^2 - V^2)$ exponential terms of equations (9) and (7). When $\gamma = 1$, the leading multiplier on the exponential term in equation (11) matches that of equation (9). And when $R = Y$, the exponent on the monomial becomes zero, leading to the form of equation (7). While the forms for $U(V)$ and $\dot{L}(V)$, obtainable from equations (2) and/or (3), are vastly different in appearance for the special and general cases, the solutions for $L(V)$ nonetheless all share a common structured form described by equation (5).

3. Choice of Model Variable

While equations (6), (8) and (10) of the previous section choose to cast the problem in terms of rod velocity and its derivatives, this is by no means the only option. Through equations (2) and (3), V may be algebraically expressed in terms of U or $\dot{L}$. Therefore, instead of expressing rod length as $L = L(V)$ in equation (11), alternate expressions of the result, given as $L = L(U)$ or $L = L(\dot{L})$ may be obtained as:

$$\frac{L}{L_0} = \left(\frac{U}{U_0} \cdot \frac{1+\sqrt{1+2(R-Y)/(\gamma \rho_R U^2)}}{1+\sqrt{1+2(R-Y)/(\gamma \rho_R U_0^2)}} \right)^{\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \cdot \exp\left[\frac{-\rho_R \sqrt{\gamma}(1+\sqrt{\gamma})}{2Y} \times \left(U_0^2 \cdot \frac{\sqrt{1+2(R-Y)/(\gamma \rho_R U_0^2)}+\sqrt{\gamma}}{1+\sqrt{\gamma}} - U^2 \cdot \frac{\sqrt{1+2(R-Y)/(\gamma \rho_R U^2)}+\sqrt{\gamma}}{1+\sqrt{\gamma}} \right) \right], \tag{12}$$

$$\frac{L}{L_0} = \left(\frac{\dot{L}}{\dot{L}_0} \cdot \frac{1+\sqrt{1-2(R-Y)/(\rho_R \dot{L}^2)}}{1+\sqrt{1-2(R-Y)/(\rho_R \dot{L}_0^2)}} \right)^{\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \cdot \exp\left[\frac{-\rho_R (1+\sqrt{\gamma})}{2Y\sqrt{\gamma}} \times \left(\dot{L}_0^2 \cdot \frac{\sqrt{1-2(R-Y)/(\rho_R \dot{L}_0^2)}+\sqrt{\gamma}}{1+\sqrt{\gamma}} - \dot{L}^2 \cdot \frac{\sqrt{1-2(R-Y)/(\rho_R \dot{L}^2)}+\sqrt{\gamma}}{1+\sqrt{\gamma}} \right) \right] . \tag{13}$$

In the derivation of these and subsequent relations, there are several closely related, algebraic expressions that can facilitate expression and/or transformation of results. These include:

$$\sqrt{\gamma}\, U - \dot{L} = \left[\sqrt{\gamma}\, V + \sqrt{\gamma V^2 + 2(1-\gamma)(R-Y)/\rho_R} \right] / \left(1+\sqrt{\gamma}\right) ; \tag{14}$$

$$\sqrt{\gamma}\, U + \dot{L} = \left[\sqrt{\gamma}\, V - \sqrt{\gamma V^2 + 2(1-\gamma)(R-Y)/\rho_R} \right] / \left(1-\sqrt{\gamma}\right) ; \tag{15}$$

$$\dot{L} = \left[\gamma V - \sqrt{\gamma V^2 + 2(1-\gamma)(R-Y)/\rho_R} \right] / \left(1-\gamma\right) ; \tag{16}$$

$$U = \left[V - \sqrt{\gamma V^2 + 2(1-\gamma)(R-Y)/\rho_R} \right] / \left(1-\gamma\right) . \tag{17}$$

4. Model-Variable Transformation

The complications of having the model variable V, U, or $\dot{L}$ under the square root for the general case of equation (11), (12), or (13), respectively, may be circumvented with the selection of a mathematically more "natural" variable than the velocity V, U, or $\dot{L}$. Looking to equation (5) for guidance, success has been found in

$$\Phi = \frac{\sqrt{\gamma}U - \dot{L}}{\sqrt{|\Sigma|}} \quad , \tag{18}$$

where the constant Σ is defined as $2(R - Y)/\rho_R$. The variable Φ, proportional to the expression of equation (14), is always nonnegative and follows somewhat the trend of rod velocity V (it actually equals $V/\sqrt{|\Sigma|}$ when $\gamma = 1$). Not surprisingly, Φ is also proportional to $\sqrt{z}$, which was the key transformation variable employed in the original derivation [3]. The key benefit to using the Φ transformation is that $\dot{L}$ and U, rather than requiring square root terms as did equations (16) and (17) when expressed in V, may be expressed in more simply in terms of Φ as

$$\dot{L} = -\frac{\sqrt{|\Sigma|}}{2}\left(\Phi + \mathrm{sgn}(\Sigma)\frac{1}{\Phi}\right) , \tag{19}$$

and

$$U = \frac{\sqrt{|\Sigma|}}{2\sqrt{\gamma}}\left(\Phi - \mathrm{sgn}(\Sigma)\frac{1}{\Phi}\right) , \tag{20}$$

where the signum function, $\mathrm{sgn}(x)$, denotes the sign of the argument [$\mathrm{sgn}(x) = x/|x|$ for $x \neq 0$, and $\mathrm{sgn}(x) = 0$ for $x = 0$], in this case the sign of Σ. The rod velocity, V, may also be obtained directly, by substituting these expressions into equation (3):

$$V = \frac{\sqrt{|\Sigma|}}{2\sqrt{\gamma}}\left((\sqrt{\gamma}+1)\Phi + \mathrm{sgn}(\Sigma)\,\frac{(\sqrt{\gamma}-1)}{\Phi}\right) . \tag{21}$$

When Φ is used in preference to rod velocity V as the independent variable, the governing equation (5) leads to the following expression:

$$\frac{L}{L_0} = \left(\frac{\Phi}{\Phi_0}\right)^{\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \exp\left\{-\frac{1}{4\sqrt{\gamma}}\left|\frac{R}{Y}-1\right|\left[\left((\sqrt{\gamma}+1)\Phi_0^2 + \frac{(\sqrt{\gamma}-1)}{\Phi_0^2}\right) - \left((\sqrt{\gamma}+1)\Phi^2 + \frac{(\sqrt{\gamma}-1)}{\Phi^2}\right)\right]\right\} . \tag{22}$$

With minimal rearrangement, the variable Φ can be made to appear always in squared form. It is for this reason that Walters and Segletes [3] selected their transformation variable, z, proportional to Φ^2. We will do the same here, though with a different proportionality constant, so that

$$z = \Phi^2 \cdot \sqrt{\frac{\sqrt{\gamma}+1}{\left|\sqrt{\gamma}-1\right|}} \quad . \tag{23}$$

By so doing, the expression for residual rod length, equation (22), becomes

$$\frac{L}{L_0} = (z/z_0)^{\frac{1}{2\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \exp\left\{-\frac{\sqrt{|\gamma-1|}}{4\sqrt{\gamma}}\left|\frac{R}{Y}-1\right|\left[(z_0 \pm 1/z_0)-(z \pm 1/z)\right]\right\}, \tag{24}$$

where the conditional operators in equation (24) are chosen as "+" for $\gamma > 1$ and "–" for $\gamma < 1$, and

$$\sqrt{z} = \left(\frac{\sqrt{\gamma}+1}{\left|\sqrt{\gamma}-1\right|\Sigma^2}\right)^{1/4} \cdot \left(\sqrt{\gamma}U - \dot{L}\right) \;\; = \;\; \frac{\sqrt{\gamma}V + \sqrt{\gamma V^2 + (1-\gamma)\Sigma}}{\left[\left|\sqrt{\gamma}-1\right|(\sqrt{\gamma}+1)^3\ \Sigma^2\right]^{1/4}} \quad . \tag{25}$$

Like equation (11), the result given by equation (24) expresses rod length in terms of a single independent variable, in this case z. The advantage of equation (24) over equation (11) is in removing the model variable from under a radical. The choice of a proportionality constant different than that used in the prior work [3], when defining z, provides a result that reduces the number of constant parameters in the exponent. More importantly, however, the appearance in the exponential of the model variable in the specific form of $(z \pm 1/z)$ will greatly expedite the evaluation of rod penetration, as will be subsequently shown.

5. Penetration

The evaluation of penetration by way of integrating equation (4) may be transformed with equation (1) to give

$$P = \int_0^t U\,dt = -\frac{1}{\dot{V}_0}\int_V^{V_0} \frac{L}{L_0} U\ dV\ . \tag{26}$$

The particular functional forms for L and U will govern the form of the solution.

5.1 *R = Y*

Penetration may be directly evaluated in closed form for the simple case of $R=Y$, wherein L is given by equation (7), and U is proportional to V throughout the penetration event. In this case, the final penetration (given by P_f as U and V approach zero) is

$$R = Y:\ \ P_f = \frac{L_0}{\sqrt{\gamma}}\left(1 - \exp\left[\frac{-\rho_R\sqrt{\gamma}}{2Y(1+\sqrt{\gamma})}V_0^2\right]\right) . \tag{27}$$

5.2 $\gamma = 1$

For the $\gamma = 1$ special case, where the penetration velocity U is given algebraically by $U = (V - \Sigma/V)/2$, the penetration may be calculated, per equation (26), in closed form if the value of the V exponent in equation (9), given as $(R - Y)/Y$, is an even integer (i.e., R/Y is an odd integer). For these limited cases, integration by parts permits the problem reduction according to one of the following two recursion rules:

$$\int (V^2)^a e^{bV^2} dV^2 = \frac{1}{b}(V^2)^a e^{bV^2} - \frac{a}{b}\int (V^2)^{a-1} e^{bV^2} dV^2 \ \ , \ (a > 0) \ ; \tag{28a}$$

$$\int (V^2)^a e^{bV^2} dV^2 = \frac{1}{(a+1)}(V^2)^{a+1} e^{bV^2} - \frac{b}{(a+1)}\int (V^2)^{a+1} e^{bV^2} dV^2 \ \ , (a < 0) \ ; \tag{28b}$$

which is repeated until the reduced exponent on V^2 becomes zero, whereupon the process terminates with the rule

$$\int e^{bV^2} dV^2 = e^{bV^2} / b \, . \tag{29}$$

For other cases without the appropriate integral exponents, a recursion-type solution, with tabulation is still plausible, in theory. The recursion rule would be applied j times until the exponent $a \pm j$ falls between zero and unity. At that point, a tabulated solution for values of a between zero and unity (along the lines of the Gamma-function solution) is used to close out the integration. With clever use of velocity-normalization, the integral from V_0^2 to V_x^2 can be broken into two integrals, each evaluated between 0 and 1. Unfortunately, the integral remains a function of two parameters, a and b. And while tabulating a function of one parameter can be an efficient solution technique, tabulating solutions for functions of two real parameters quickly become more cumbersome than a series expansion or numerically integrated solution.

As an alternative then, the penetration for the $\gamma = 1$ special case may be evaluated by way of series solution in terms of velocity. One way to achieve this is to express the penetration as

$$\gamma = 1:\ \ \frac{P}{L_0} = \left[\sum_{j=0}^{\infty} a_j \left(\frac{\rho_R V_0^2}{4Y}\right)^j\right] - \frac{L}{L_0}\left[\sum_{j=0}^{\infty} a_j \left(\frac{\rho_R V^2}{4Y}\right)^j\right], \tag{30}$$

and match the derivative of P to the terms of U, given by $U = (V - \Sigma/V)/2$. With this approach, one obtains $a_0 = -1$, $a_1 = 2/(1 + \rho_R\Sigma/4Y)$, and for the remaining terms, $a_j = -a_{j-1}/(j + \rho_R\Sigma/4Y)$. Note that $\rho_R\Sigma/4Y$ equals $(R/Y - 1)/2$. While the series terms alternate in sign, the fact that j is in the denominator of the recursion formula indicates that the rate of convergence for this solution approach should be similar to that for the exponential series. To confirm that this expression approaches the proper form for the special case when $R=Y$ (when Σ equals zero), the recursion relation is observed to then approach the series definition for the exponential, $1-2\exp[-V^2/4K]$. This series takes on the value $1 - 2(L_f/L_0)$ when evaluated at V_0, and -1 when evaluated at $V = 0$. Here, L_f is the terminal length of the rod. The final penetration, therefore, becomes $L_0 - L_f$, as expected for $R=Y$ and $\gamma = 1$.

Perhaps a more forthright approach for the evaluation of penetration for the $\gamma = 1$ special case (and less prone to the precision problems of evaluating an alternating series) is to directly integrate $LU{\cdot}dV$, per equation (26), by initially expanding the exponential term of L into a series,

$$\frac{L}{L_0}\cdot U=\frac{1}{2}\exp\left(\frac{-\rho_R V_0^2}{4Y}\right)\left(V-\frac{\Sigma}{V}\right)\left(\frac{V}{V_0}\right)^{\frac{\rho_R\Sigma}{2Y}}\left(1+\frac{\left(\rho_R V^2/4Y\right)}{1!}+\frac{\left(\rho_R V^2/4Y\right)^2}{2!}+\dots\right), \tag{31}$$

and integrating term by term to the desired level of precision. As before, $\Sigma = 2(R-Y)/\rho_R$. By integrating this expression with respect to V, per equation (26), one may obtain

$\gamma = 1$:

$$\frac{P}{L_0}=\exp\left[\frac{-\rho_R V_0^2}{4Y}\right]\left\{\sum_{j=0}^{\infty}\frac{1}{j!}\cdot\frac{j-\frac{\rho_R\Sigma}{4Y}}{j+\frac{\rho_R\Sigma}{4Y}}\cdot\left(\frac{\rho_R V_0^2}{4Y}\right)^j-\left(\frac{V^2}{V_0^2}\right)^{\frac{\rho_R\Sigma}{4Y}}\sum_{j=0}^{\infty}\frac{1}{j!}\cdot\frac{j-\frac{\rho_R\Sigma}{4Y}}{j+\frac{\rho_R\Sigma}{4Y}}\cdot\left(\frac{\rho_R V^2}{4Y}\right)^j\right\}. \tag{32}$$

5.3 General Case

In evaluating the penetration for the general case, the solution becomes more complicated but can nonetheless be made more efficient compared to the method presented in the original solution [3]. Efficiencies are achieved in several ways. The use of rod length L in the form of equation (24) retains integer-powered polynomials in the exponential term. As such, the series expansion of the exponential, by which the integrals are evaluated, does not require the evaluation of fractionally powered polynomial expansions, as did the original method [3]. But more importantly, by having transformed L into a form where the exponential argument is of the explicit form $c(z \pm 1/z)$, a method may be used to expand the exponential in an efficient way, reducing the expansion of the exponential to power n from a cost of $(n + 1)(n + 2)/2$ monomial evaluations in z, to one of $2n + 1$ evaluations in z.

The equation describing the penetration, equation (26), may be reorganized to obtain an expression in terms of the transformation variable, z,

$$P=\int_0^t U\,dt=\int_{z_0}^{z}\frac{U}{\dot V}\frac{dV}{dz}dz \quad = \quad -\frac{1}{\dot V_0}\int_{z}^{z_0}\frac{L}{L_0}U\,\frac{dV}{dz}\,dz\,. \tag{33}$$

Using equations (21) and (23), the rod velocity is expressible in terms of z as

$$V = \frac{\left(|\gamma-1|\Sigma^2\right)^{1/4}}{2\sqrt{\gamma}}\left[(\sqrt{\gamma}+1)^{1/2}\sqrt{z} + \operatorname{sgn}[(\gamma-1)\Sigma]\frac{\left|\sqrt{\gamma}-1\right|^{1/2}}{\sqrt{z}}\right], \tag{34}$$

so that dV/dz may be computed as

$$\frac{dV}{dz} = \frac{\left(|\gamma-1|\Sigma^2\right)^{1/4}}{4\sqrt{\gamma}}\left[\frac{(\sqrt{\gamma}+1)^{1/2}}{z^{1/2}} - \operatorname{sgn}[(\gamma-1)\Sigma]\frac{\left|\sqrt{\gamma}-1\right|^{1/2}}{z^{3/2}}\right]. \tag{35}$$

In a similar vein, from equations (20) and (23), U may be expressed in terms of z as

$$U = \frac{\sqrt{|\Sigma|}}{2\sqrt{\gamma}}\left[\left(\frac{\left|\sqrt{\gamma}-1\right|}{\sqrt{\gamma}+1}\right)^{1/4}\sqrt{z} - \operatorname{sgn}(\Sigma)\left(\frac{\sqrt{\gamma}+1}{\left|\sqrt{\gamma}-1\right|}\right)^{1/4}\frac{1}{\sqrt{z}}\right]. \tag{36}$$

The product, $U \cdot dV/dz$, may therefore be computed as

$$U\frac{dV}{dz} = \frac{|\gamma-1|^{1/4}|\Sigma|}{8\gamma} \times \left\{|\gamma-1|^{1/4} - \operatorname{sgn}(\Sigma)\left(\left[\frac{(\sqrt{\gamma}+1)^3}{\left|\sqrt{\gamma}-1\right|}\right]^{1/4} + \operatorname{sgn}(\gamma-1)\left[\frac{\left|\sqrt{\gamma}-1\right|^3}{(\sqrt{\gamma}+1)}\right]^{1/4}\right)\frac{1}{z} + \operatorname{sgn}(\gamma-1)\frac{|\gamma-1|^{1/4}}{z^2}\right\}, \tag{37}$$

which is of the form

$$U \cdot dV/dz = A\left(a_0 + a_1/z + a_2/z^2\right). \tag{38}$$

Substituting this result and the transformed expression for L, given by equation (24), into equation (33) allows the integral for penetration to take the form

$$P = B_P\int_z^{z_0}(a_0 + a_1/z + a_2/z^2)z^b \exp[c(z \pm 1/z)]dz\,, \tag{39}$$

where the conditional minus sign in the exponential is taken when $\gamma < 1$, and a_i, b, c, and B_P are all constants, expressible as

$$a_0 = |\gamma-1|^{1/4}, \tag{40}$$

$$a_1 = -\operatorname{sgn}(R-Y)\left(\left[\frac{(\sqrt{\gamma}+1)^3}{\left|\sqrt{\gamma}-1\right|}\right]^{1/4} + \operatorname{sgn}(\gamma-1)\left[\frac{\left|\sqrt{\gamma}-1\right|^3}{(\sqrt{\gamma}+1)}\right]^{1/4}\right) , \tag{41}$$

$$a_2 = \operatorname{sgn}(\gamma-1)\left|\gamma-1\right|^{1/4} , \tag{42}$$

$$b = \frac{1}{2\sqrt{\gamma}}\left(\frac{R}{Y}-1\right) , \tag{43}$$

$$c = \frac{1}{4}\sqrt{\frac{\left|\gamma-1\right|}{\gamma}}\left|\frac{R}{Y}-1\right| , \tag{44}$$

and

$$B_P = L_0\left|\frac{R}{Y}-1\right|\frac{\left|\gamma-1\right|^{1/4}}{4\gamma \cdot z_0^b}\exp[-c(z_0 + \operatorname{sgn}(\gamma-1)/z_0)] . \tag{45}$$

The form of equation (39) is basically identical to an intermediate step of the original solution [3], though with differently defined constants. The prior work [3] opted to transform the equation again to eliminate the leading polynomials, but did so at the expense of introducing noninteger powers into the exponential term. Then, the penetration equation was solved by expanding the exponential into a power series of $(A_1 \cdot z^g + A_2 \cdot z^{-g})^j$ terms and expanding each $(A_1 \cdot z^g + A_2 \cdot z^{-g})^j$ term into $j + 1$ monomials, using a binomial expansion. The net result of the total expansion was that, to include terms out to a power of $j = n$, a total of $(n + 1)(n + 2)/2$ monomials was generated, and then integrated term by term. With n routinely exceeding 20 to obtain the desired precision, and approaching 100 for certain initial conditions, the computational burden was substantial, though still more efficient than a numerical integration of equations (1)–(4).

While the currently proposed method still relies on a series expansion of the exponential to perform the integration, a technique permits a streamlined method for achieving the expansion. In particular, a method exists to expand the subject exponential series with the form

$$\exp[c(z \pm 1/z)] = \sum_{j=-\infty}^{\infty} C_j^{\pm} z^j , \tag{46}$$

where the C_j^+ or C_j^- coefficients are a function only of the parameter c. In particular, the C_j^- constants are given by evaluations of Bessel functions of the first kind, such that $C_j^- = J_j(2c)$. The C_j^+ constants, by contrast, are given by modified Bessel functions of the first kind, such that $C_j^+ = I_j(2c)$. The expansion using the form of equation (46), to include terms of power $z^{\pm n}$, requires the evaluation of only $2n + 1$ monomials in z, and therefore represents a significant improvement over the method previously employed [3], which required the evaluation of $(n + 1)(n + 2)/2$ monomials in z for identical precision.

While there is an overhead associated with the evaluation of the $C_j^\pm$ parameters, given by the converging series that defines the Bessel functions for integer order,

$$C_j^\pm = \begin{cases} \displaystyle\sum_{i=0}^{\infty} \frac{(\pm 1)^i c^{2i+j}}{i!\,(i+j)!} & , j \geq 0 \\ \\ (\pm 1)^j C_{-j}^\pm & , j < 0 \end{cases} , \tag{47}$$

the parameter c is fixed by the initial conditions (material properties) of the penetration problem. As such, the C_j^+ or C_j^- terms may be calculated once at the onset of the analysis, regardless of how many z values (i.e., velocities) for which the solution needs evaluation. Furthermore, there exists a recursive technique for evaluating the $C_j^\pm$ parameters of equation (47), based on the recursions

$$\frac{C_j^+}{C_{j-1}^+} = \frac{1}{\dfrac{j}{c} + \dfrac{C_{j+1}^+}{C_j^+}} , \tag{48a}$$

and

$$\frac{C_j^-}{C_{j-1}^-} = \frac{1}{\dfrac{j}{c} - \dfrac{C_{j+1}^-}{C_j^-}} , \tag{48b}$$

which thereby offers further computational savings.

The integration for penetration is, thus, finally achieved by employing this optimized expansion and integrating term by term and evaluating at the desired limits. When b is not an integer, which is the typical case, the result may be expressed as

$$\text{General case:} \quad P = B_P \sum_{j=-\infty}^{\infty} \left(a_0 C_{j-1}^\pm + a_1 C_j^\pm + a_2 C_{j+1}^\pm \right) \frac{z^{j+b}}{j+b} \Bigg|_z^{z_0} , \tag{49}$$

where the C^+ terms are used when $\gamma > 1$ and the C^- terms are used when $\gamma < 1$. For the case when b is an integer, the single term of equation (49) that would otherwise produce a zero in the denominator (i.e., the term for which $j = -b$) originated from a $1/z$ integration and would actually have produced, upon integration, the logarithmic term $\ln(z)$, instead of $z^{j+b}/(j+b)$.

6. Implicit Time

Though these solutions for $L(V)$ and $P(V)$ bypass the intermediate evaluation of $V(t)$, the penetration variables may, if needed, be implicitly expressed in terms of time, by integration of $L(V)$,

$$t = \int_{V_0}^{V} \frac{dV}{\dot{V}} = -\frac{1}{\dot{V}_0} \int_{V}^{V_0} \frac{L}{L_0} dV \,, \tag{50}$$

in order to obtain $t(V)$. As in the case of penetration, a closed-form solution to equation (50) will be possible only for the special case of $\gamma = 1$ and then only when $(R - Y)/Y$ is an odd integer (i.e., R/Y is even). In all other cases, the integration of equation (50) will take the form of a series solution. Of the several ways to obtain a series integration of the special case solutions, a power-series expansion is preferable to a repeated integration-by-parts solution because it avoids an alternating series, for the case when the "c" constant associated with the $\exp[-c(V_0^2 - V^2)]$ term is positive. Such is always the case for penetration problems. Both the $R = Y$ and $\gamma = 1$ special cases can be reduced to an integral of the form

$$\int_0^a V^b \exp(cV^2) dV = a^{b+1} \sum_{i=0}^{\infty} \frac{(ca^2)^i}{i!(2i+b+1)} \quad . \tag{51}$$

Thus, the special-case solutions for $t(V)$ may be evaluated as

$R = Y$:

$$t = \frac{\rho_R L_0 V_0}{Y} \exp\left[\frac{-\rho_R \sqrt{\gamma} V_0^2}{2Y(1+\sqrt{\gamma})}\right] \left[\sum_{i=0}^{\infty} \frac{1}{i!(2i+1)} \left(\frac{\rho_R \sqrt{\gamma} V_0^2}{2Y(1+\sqrt{\gamma})}\right)^i - \frac{V}{V_0} \sum_{i=0}^{\infty} \frac{1}{i!(2i+1)} \left(\frac{\rho_R \sqrt{\gamma} V^2}{2Y(1+\sqrt{\gamma})}\right)^i \right], \tag{52}$$

$\gamma = 1$:

$$t = \frac{\rho_R L_0 V_0}{Y} \exp\left[\frac{-\rho_R V_0^2}{4Y}\right] \left[\sum_{i=0}^{\infty} \frac{1}{i!(2i+R/Y)} \left(\frac{\rho_R V_0^2}{4Y}\right)^i - \left(\frac{V}{V_0}\right)^{R/Y} \sum_{i=0}^{\infty} \frac{1}{i!(2i+R/Y)} \left(\frac{\rho_R V^2}{4Y}\right)^i \right]. \tag{53}$$

For the general case, a solution is most profitably obtained in a manner analogous to the penetration evaluation, in which a transformation to z facilitates a streamlined series solution:

$$t = \int_{V_0}^{V} \frac{dV}{\dot{V}} = -\frac{1}{\dot{V}_0} \int_{z}^{z_0} \frac{L}{L_0} \frac{dV}{dz} dz \,. \tag{54}$$

This integration may be staged through the substitution of equations (24) and (35), to give the following form:

$$t = B_t \int_z^{z_0} (d_0/z^{1/2} + d_1/z^{3/2}) z^b \exp[c(z \pm 1/z)] dz\,, \tag{55}$$

where the conditional minus sign is taken when $\gamma < 1$. Here, b and c are defined as before, by equations (43) and (44), while

$$d_0 = (\sqrt{\gamma} + 1)^{1/2}\,, \tag{56}$$

$$d_1 = -\mathrm{sgn}[(\gamma - 1)(R - Y)] \left| \sqrt{\gamma} - 1 \right|^{1/2}\,, \tag{57}$$

and

$$B_t = L_0 \sqrt{\frac{\rho_R}{Y}} \left(\frac{1}{8\gamma} \cdot \left| \frac{R}{Y} - 1 \right| \right)^{1/2} \frac{\left| \gamma - 1 \right|^{1/4}}{z_0^b} \exp[-c(z_0 + \mathrm{sgn}(\gamma - 1)/z_0)]\,. \tag{58}$$

By using a method analogous to that in equations (46)–(49) and with the same definitions for $C_j^{\pm}$ [given by equation (47), where the "+" solution applies for $\gamma > 1$, and the "–" solution for $\gamma < 1$], the expression for t given by equation (55) may be expanded in a series as

$$\text{General case:}\quad t = B_t \sum_{j=-\infty}^{\infty} \left(d_0 C_{j-1}^{\pm} + d_1 C_j^{\pm} \right) \left. \frac{z^{j+b-1/2}}{j + b - 1/2} \right|_z^{z_0}\,. \tag{59}$$

Like equation (49), there is one exception to the general validity of this result, specifically for the case when b is precisely a half-integer. If and only if this is the case, a single term of equation (59) will require modification: namely, the term for which $j + b - 1/2$ exactly equals zero, originating from a $1/z$ integration. This integration would, for this one term only, rightfully have produced a $\ln(z)$ term, instead of $z^{j+b-1/2}/(j+b-1/2)$. As with the evaluation of penetration, the summation of equation (59) is carried out for j over some finite range from $-n$ to $+n$ so as to achieve the desired level of precision.

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7. Terminal Rod Length, etc.

The "terminal" rod length may be ascertained for the various solution cases [from equations (7), (9) or (11)], by setting V to its terminal value, $V_x = \sqrt{\Sigma}$ for the case of $R > Y$ and $V_x = \sqrt{-\Sigma/\gamma}$ for $R < Y$, with the parameter Σ given by $\Sigma = 2(R - Y)/\rho_R$. When $R > Y$, this termination corresponds to the point where $U = 0$, when the penetration ceases (though the rod may continue to erode thereafter). For $R < Y$, the termination corresponds to the point where $\dot{L} = 0$, when the rod erosion ceases (though the rod may continue to penetrate as a rigid body thereafter). This terminal state, denoted with the subscript "x," corresponds not to the end of the ballistic event, but rather to the time at which the governing equations (1)–(4) cease to apply. In those governing equations, developed for the case of a simultaneously eroding rod and target, the subscript "x" condition corresponds to the moment at which either the rod or the target stops eroding. In general, these two conditions do not occur simultaneously. The rod length (normalized) at the terminal state "x" for the various cases is expressible as:

$$R=Y\text{:}\quad \frac{L_x}{L_0} = \exp\left[\frac{-\rho_R\sqrt{\gamma}}{2Y(1+\sqrt{\gamma})}V_0^2\right], \tag{60}$$

$$\gamma = 1\text{:}\quad \frac{L_x}{L_0} = \left[\frac{V_0^2}{|\Sigma|}\right]^{-\frac{1}{2}\left(\frac{R}{Y}-1\right)} \exp\left[-\frac{1}{2}\left|\frac{R}{Y}-1\right|\left(\frac{V_0^2}{|\Sigma|}-1\right)\right], \tag{61}$$

General case:

$$\frac{L_x}{L_0} = \left(\frac{\sqrt{\gamma}(V_0/\sqrt{|\Sigma|}) + \sqrt{\gamma(V_0/\sqrt{|\Sigma|})^2 + \operatorname{sgn}(\Sigma)(1-\gamma)}}{1+\sqrt{\gamma}}\right)^{-\frac{1}{\sqrt{\gamma}}\left(\frac{R}{Y}-1\right)} \times$$
$$\exp\left[-\left|\frac{R}{Y}-1\right|\left(\frac{(V_0/\sqrt{|\Sigma|})\sqrt{\gamma(V_0/\sqrt{|\Sigma|})^2 + \operatorname{sgn}(\Sigma)(1-\gamma)} - \gamma(V_0/\sqrt{|\Sigma|})^2}{1-\gamma} - \frac{1+\operatorname{sgn}(\Sigma)}{2}\right)\right] . \tag{62}$$

For cases where $R > Y$, this terminal length corresponds to that length of rod as of the moment that penetration ceases. For $R < Y$, this is the rod length at the onset of rigid-body penetration.

Terminal values (at state "x") for penetration and time may likewise be obtained by evaluating the respective relations [equations (30), (32), or (49) for penetration and equations (52), (53), or (59) for time] with the substitution of $V = V_x$ [and $L = L_x$ in the case of equation (30)]. Their presentation is omitted, however, because these relations are summations and not in closed form like those for residual length previously given. As such, there is little clarity of reduction gained in restating these earlier equations with the $V = V_x$ substitution in place.

8. Residual Erosion/Penetration Behaviors

Equations (1) and (2) are valid only while there is simultaneous target penetration and rod erosion. Except for the special case of $R = Y$, $\dot{L}$ and U will not simultaneously approach zero. In the general case then, the physical event will continue with either residual rod erosion following the cessation of penetration (when $R > Y$) or residual rigid body penetration following the cessation of rod erosion (when $R < Y$). These afterflow events are amenable to closed-form analytical solution. Continuing to denote the state at this transition point (the moment of transition to either rigid target or rigid rod) with the use of the subscript "*x*," the absolute final state, when the rod velocity itself finally reaches zero, will be denoted with the subscript "*f*." Recall that $V_x = \sqrt{\Sigma}$ when $R > Y$, while $V_x = \sqrt{-\Sigma/\gamma}$ when $R < Y$, where $\Sigma = 2(R - Y)/\rho_R$.

8.1 Residual Rod Erosion

For the case of R > Y, the target becomes rigid while rod erosion continues. To deal with this, equation (2) is discarded and is substituted with the constraint U = 0. The kinematic constraint of equation (3) becomes, as a result, $V = -\dot{L}$. Equation (1) remains valid for the eroding-rod case. Solving equation (1) for L, differentiating, and substituting the revised kinematic constraint to eliminate $\dot{L}$, one obtains as the governing equation

$$V\dot{V}^2 = -(Y/\rho_R)\ddot{V} . \tag{63}$$

This may be integrated to obtain $\dot{V}$ in terms of V, whereupon equation (1) may be used to eliminate $\dot{V}$ in favor of L. The result (as a function of V) is that

$$L = L_x \exp\left[\frac{-\rho_R}{2Y}(V_x^2 - V^2)\right]. \tag{64}$$

Evaluating the penetration and rod length at the final state (where $V = 0$), one obtains $P_f = P_x$ and

$$L_f = L_x \exp\left[\frac{-\rho_R V_x^2}{2Y}\right]. \tag{65}$$

Because of the similarity between the governing equation here, equation (63), and the special case $R = Y$ governing equation, equation (6), the duration of this residual-erosion phase of the rod may likewise be calculated with the same series-solution form used to calculate event duration for the special cases, described by equation (51). Use of this form leads to

$$t - t_x = \frac{\rho_R L_x V_x}{Y}\exp\left(\frac{-\rho_R V_x^2}{2Y}\right)\left[\sum_{i=0}^{\infty}\frac{1}{i!(2i+1)}\left(\frac{\rho_R V_x^2}{2Y}\right)^i - \frac{V}{V_x}\sum_{i=0}^{\infty}\frac{1}{i!(2i+1)}\left(\frac{\rho_R V^2}{2Y}\right)^i\right], \tag{66}$$

which, as V approaches zero, becomes the following result:

$$t_f - t_x = \frac{\rho_R L_x V_x}{Y}\exp\left(\frac{-\rho_R V_x^2}{2Y}\right)\sum_{i=0}^{\infty}\frac{1}{i!(2i+1)}\left(\frac{\rho_R V_x^2}{2Y}\right)^i . \tag{67}$$

8.2 Residual Rigid-Body Penetration

For the alternate case of $R < Y$, a state of rigid-body penetration is reached after the rod erosion ceases. As before, equation (2) is discarded and is substituted with the constraint $\dot{L} = 0$. The kinematic constraint (3) becomes, as a result, $V = U$. However, there is one additional modification required to the governing equations. In particular, the force causing the rod deceleration in equation (1) is no longer Y, since the rod is no longer in a plastic state. Rather, it is a diminished stress state applied by the pressure head and resistance of the target, $1/2\rho_T U^2 + R$. But since, kinematically, $V = U$ and L remains fixed at L_x, the rod deceleration equation becomes

$$L_x \dot{V} = -(1/2\rho_T V^2 + R)/\rho_R \, . \tag{68}$$

There is no algebraic relation between P and $\dot{V}$ analogous to that which equation (1) affords between L and $\dot{V}$. Therefore, this equation will be solved by separating the variables V and t, as follows:

$$\frac{2L_x}{\gamma} \cdot \frac{dV}{2R/\rho_T + V^2} = -dt \, . \tag{69}$$

This may be solved as

$$V = U = \sqrt{\frac{2R}{\rho_T}} \tan\left[\frac{\gamma}{L_x} \sqrt{\frac{R}{2\rho_T}} (t_x - t) + \tan^{-1}\left(V_x \sqrt{\frac{\rho_T}{2R}} \right) \right]. \tag{70}$$

The final time, at which the velocity drops to zero, is found to be

$$t_f = t_x + \frac{L_x}{\gamma} \sqrt{\frac{2\rho_T}{R}} \tan^{-1}\left(V_x \sqrt{\frac{\rho_T}{2R}} \right). \tag{71}$$

The expression for U, which is equation (70), may be integrated one more time to obtain the differential penetration that occurs during the afterflow phase. One obtains

$$P - P_x = \frac{2L_x}{\gamma} \left\{ \log\cos\left[\frac{\gamma}{L_x} \sqrt{\frac{R}{2\rho_T}} (t_f - t) \right] - \log\cos\tan^{-1}\left(V_x \sqrt{\frac{\rho_T}{2R}} \right) \right\}. \tag{72}$$

When evaluated at $t = t_f$, and employing some trigonometric substitutions, the final result is that $L_f = L_x$ and the afterflow penetration is

$$P_f - P_x = \frac{L_x}{\gamma} \log\left(1 + \frac{\rho_T V_x^2}{2R} \right). \tag{73}$$

9. Conclusions

This report presents updated results related to the exact solution of the long-rod penetration equations, formulated by Alekseevskii [1] and Tate [2], and first solved by Walters and Segletes [3]. While the original solution [3] is accurate and comprehensive, there have been a number of improvements or enhancements, both to the presentation and the solution approach.

Equation (5) is a concise analytical presentation of rod length as a function of rod velocity, valid for both special and general cases, providing an enhanced sense for the terms that drive the analytical solution. Equations (6)–(11) compare and contrast the special- and general-case analytical solutions, while equations (12) and (13) present the result in terms of an alternate model variable. The key independent variable transformation (to z), unexplained but indispensable to the original solution, is herein developed more fully and much of its mystery is thereby uncloaked. Further, its expression is slightly altered from the original solution, resulting, by comparison, in a form amenable to a highly streamlined series solution for penetration $P(z)$, as equation (49), or implicit time, $t(z)$, as equation (59).

Not only are results derived to the point where the penetration equations cease validity, but extensions to the original solution are presented, which account for the period of rigid-body penetration or rigid-target rod erosion that follows the period of eroding-body penetration addressed by the original penetration equations.

While not taking anything from the original solution of Walters and Segletes [3], the current work offers enhanced appreciation and understanding of the original effort, as well as extensions to the original work. Finally, the streamlined techniques presented herein make any implementation of the solution significantly more efficient than the originally offered solution technique.

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The exact solution to the long-rod penetration equations is revisited, in search of improvements to the solution efficiency, while simultaneously enhancing the understanding of the physical parameters that drive the solution. Substantial improvements are offered in these areas. The presentation of the solution is simplified in a way that more tightly unifies the special- and general-case solutions to the problem. Added computational efficiencies are obtained by expressing the general-case solution for penetration and implicit time in terms of a series of Bessel functions. Other extensions and efficiencies are addressed, as well.

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