



**Diffraction Effects in the Determination of Damage
Thresholds Using Focused Top Hat Beams**

by Timothy M. Pritchett

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1. Introduction

The familiar intensity distribution associated with Fraunhofer diffraction from a slit or aperture may be observed not only in the far field, but also in the focal plane of a well-corrected lens. Accordingly, an accurate determination of damage thresholds for focal plane elements must correctly account for diffractive effects.

Of course, a reported value of the fluence (incident energy per unit area) at which an optical component experiences damage is truly meaningful only if that fluence is appropriately defined. Depending on, for instance, the size or thermal properties of the component in question, the peak fluence to which the component is subject may prove a less useful measure of the damage threshold than the average fluence impinging on the component or on some portion thereof. Thus, in cases in which an average value of the fluence is deemed to provide the most meaningful measure of the damage threshold, one must exercise a certain amount of care in the choice of convention regarding the area over which average fluences are to be calculated.

In this technical note, we review the physics of Fraunhofer diffraction from a uniformly illuminated circular aperture (a top hat beam) and describe the computation of peak and average fluences in the presence of diffractive effects.

2. Diffraction From a Circular Aperture

We consider the situation illustrated in Figure 1: a lens of focal length L situated in a circular aperture of radius a , upon which a planar wave front is incident. (In practice, the incident radiation is provided by a high-quality Gaussian laser beam that overfills the aperture to the extent that the wave front can be well approximated as planar.) The incident radiation is monochromatic with wavelength λ ; $k = 2\pi/\lambda$ is the wavenumber. Let ρ denote the radial coordinate in the image plane and for later convenience define $w = \sin \theta = \rho/\sqrt{\rho^2 + L^2}$.

The irradiance profile in the image plane is given by the familiar result of Airy,¹

$$I(\sin \theta) = I(0) \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2, \quad (1)$$

indicated by the black line in Figure 2. Dark rings are observed when $ka \sin \theta$, the argument of the Bessel function in equation 1, assumes a value associated with a root, i.e., when $ka \sin \theta = 3.832, 7.016, 10.174, \dots$, or equivalently, when $\sin \theta = 0.610 \lambda/a, 1.116 \lambda/a, 1.619 \lambda/a$, etc. For the innermost dark ring, $\theta \ll 1$, so $\sin \theta \approx \tan \theta = \rho/L$, and one can write the following

¹Airy, G. B. *Trans. Camb. Phil. Soc.* **1835**, 5, 283.

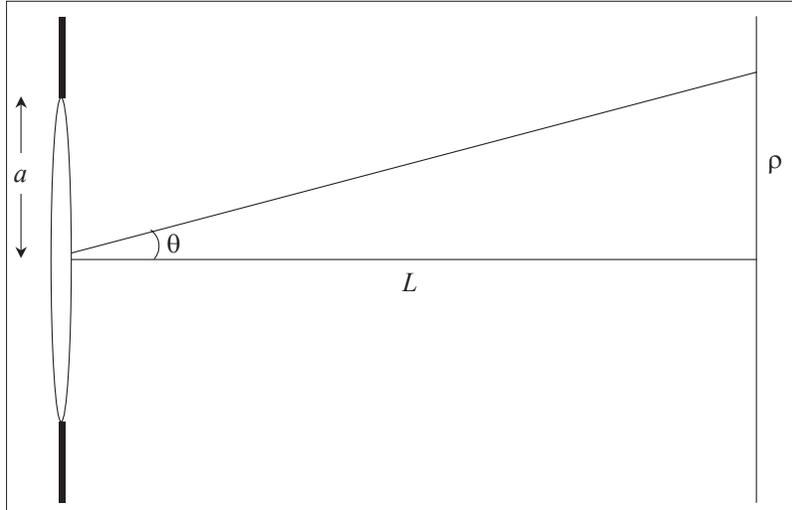


Figure 1. Diffraction from a circular aperture.

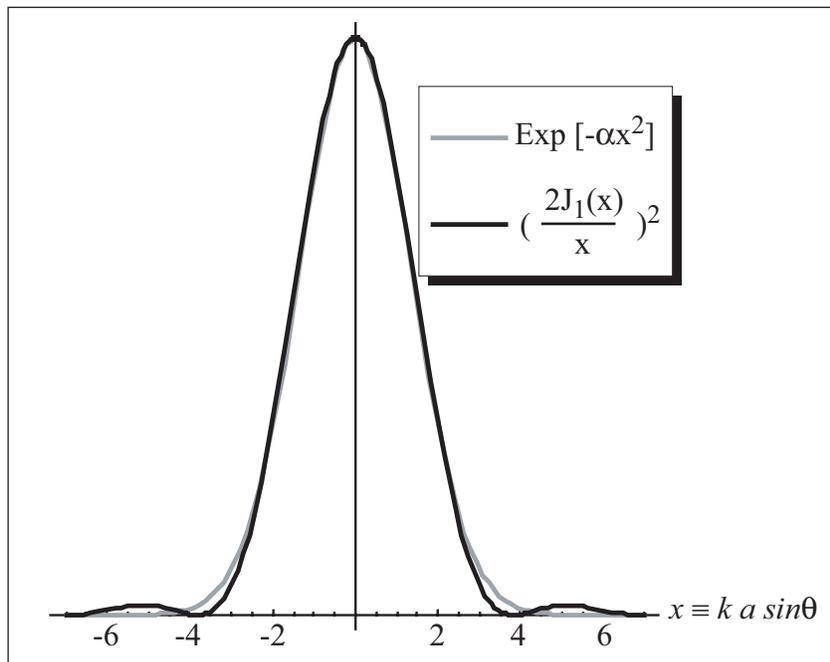


Figure 2. The Airy pattern (with “best fit” Gaussian).

well-known expression for the radius of the first dark ring in terms of the ratio $\frac{L}{2a}$, the f -number ($f/\#$) of the system: $\rho_1 \approx 0.610\lambda \frac{L}{a} = 1.22\lambda (f/\#)$.

Using a very general argument involving energy conservation, one can show that the on-axis irradiance $I(0)$ is, to a very good approximation, given by the following simple formula:

$$I(0) = \frac{\pi a^2 P}{\lambda^2 L^2}, \quad (2)$$

in which P is the total power passing through the aperture. Equation 2 is obtained from an expression for the irradiance that is derived with the restriction that $\sin \theta \ll 1$. This expression is integrated over the image plane. However, the error made in extending the integration over the entire image plane is negligible, since the integral is dominated by the contribution from the paraxial region, in which the restriction holds; at large angles θ from the optic axis, the irradiance is vanishingly small.²

Integration of equation 1 over all time yields the total fluence $F(\sin \theta)$, which is simply the product of the irradiance (equation 1), the pulse width, and a dimensionless constant dependent on the pulse shape. For instance, for a single pulse that is Gaussian in time with $\text{HW}(1/e^2)$ pulse width τ , the fluence is given by $F(\sin \theta) = \sqrt{\pi} \tau I(\sin \theta)$, while for a square pulse of duration T , it is simply $T I(\sin \theta)$. The spatial profile of the fluence is identical to that of the irradiance:

$$F(\sin \theta) = F(0) \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \quad (3)$$

and $F(0)$, the fluence at the focal point, is

$$F(0) = \frac{\pi a^2 E}{\lambda^2 L^2} = \frac{\pi E}{4\lambda^2 (f/\#)^2}. \quad (4)$$

In equation 4, E represents the total energy passing through the aperture and $f/\#$ is the f -number of the system, $1/2a$.

The appearance of λ^2 in the denominator of equations 2 and 4 might at first seem puzzling. To make the presence of this factor more plausible, one might recall that the effects of diffraction from the aperture diminish in importance at short wavelengths. Thus, for small λ , one would expect to see an increase in the amount of energy that the lens concentrates at focus, just as equation 4 predicts.

3. Encircled Energy and Average Fluence

In the determination of damage thresholds for certain types of focal plane elements, the peak fluence $F(0)$ may prove less useful than the *average fluence* incident on a given region. This is defined simply as the ratio of the total energy deposited in the region to the area of the region. In the present context, in which the setup is cylindrically symmetric about the optic axis, it is natural to consider a disk-shaped region of the image plane with the focal point at the center.

² Born, M.; Wolf, E. *Principles of Optics*, 6th ed.; Cambridge University Press: Cambridge, UK; 1980; p 386.

The average fluence incident on a disk of radius ρ_0 (which, seen from a point at the center of the aperture, subtends the angular radius $w_0 = \rho_0 / \sqrt{\rho_0^2 + L^2}$) is

$$\bar{F}(w_0) = \frac{E(w_0)}{\pi\rho_0^2}, \quad (5)$$

where $E(w_0)$, the total energy deposited in the disk, is the *encircled energy* in the solid angle subtended by the disk, as seen from the center of the aperture. $E(w_0)$ is the integral of the fluence over the disk.

$$E(w_0) = 2\pi \int_0^{\rho_0} F(w)\rho d\rho = 2\pi F(0) \frac{L^2}{k^2 a^2} \int_0^{w_0} \frac{dw}{1-w^2} \frac{[2J_1(kaw)]^2}{w}. \quad (6)$$

Since $w^2 < 1$, the factor $[1-w^2]^{-1}$ in the integrand of the latter equality can be replaced by the geometric series $\sum w^{2n}$. Doing so, and substituting for $F(0)$ from equation 4, one obtains

$$\begin{aligned} E(w_0) &= \frac{1}{2} E \sum_{n=0}^{\infty} \int_0^{w_0} dw w^{2n-1} [2J_1(kaw)]^2 \\ &= 2E \left\{ \int_0^{w_0} dw \frac{J_1^2(kaw)}{w} + \int_0^{w_0} dw w J_1^2(kaw) + \dots \right\}. \end{aligned} \quad (7)$$

The integrations may be performed by making use of the recursion relations for the Bessel functions. The result, expressed as the fraction $\varepsilon(w_0)$ of the total incident energy contained within a circle in the image plane that is centered at the focal point and, seen from the center of the aperture, subtends the solid angle $2\pi w_0$, is

$$\begin{aligned} \varepsilon(w_0) \equiv \frac{E(w_0)}{E} &= 1 - J_0^2(kaw_0) - J_1^2(kaw_0) \\ &+ w_0^2 [J_1^2(kaw_0) - J_0(kaw_0)J_2(kaw_0)] + o(w_0^4 [kaw_0]^2). \end{aligned} \quad (8)$$

To the best of our knowledge, this is a new result, but it is of little practical interest, since the version of this formula published by Lord Rayleigh in 1881,³

$$\varepsilon(w_0) \approx 1 - J_0^2(kaw_0) - J_1^2(kaw_0), \quad (9)$$

though valid only to zeroth order in w_0 , suffices completely for most applications. It is plotted in Figure 3.

³ Rayleigh, J. W. S. *Phil. Mag.* **1881**, *11*, 214.

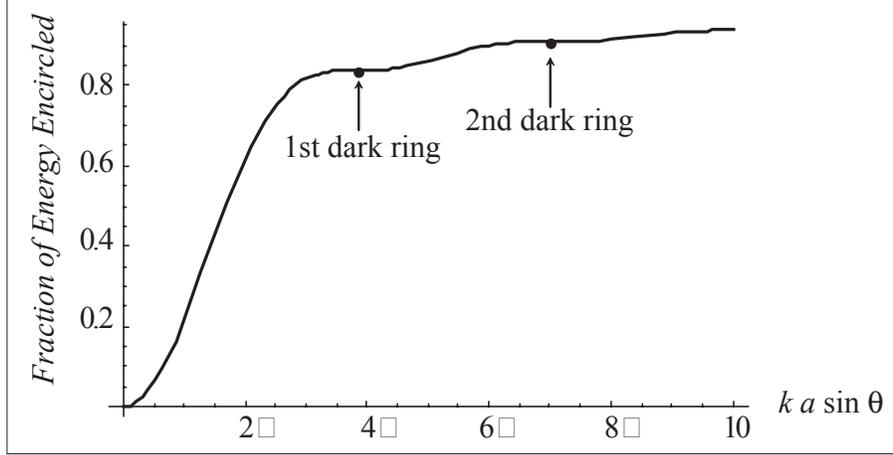


Figure 3. Fraction of total energy contained in a disk in the focal plane that is centered at focus and subtends solid angle $2\pi \sin \theta$ at the center of the aperture.

Written in terms of $\varepsilon(w_0)$, the average fluence is

$$\bar{F}(w_0) = \frac{E \varepsilon(w_0)}{\pi \rho_0^2} = E \varepsilon(w_0) \frac{1 - w_0^2}{\pi w_0^2 L^2} \approx \frac{E \varepsilon(w_0)}{\pi w_0^2 L^2}, \quad (10)$$

where the last expression is valid for $\theta \ll 1$. Here, the disk radius ρ_0 (or equivalently, w_0 , the angular radius subtended by the disk) is arbitrary, and various workers may make different choices for the size of their “standard disk.” A particularly natural choice is to take the disk to be the central bright disk of the Airy pattern, the region bounded by the first dark ring at $w_0 = 0.610 \lambda / a$. The central Airy disk contains 84% of the energy passing through the aperture. Alternately, one might choose w_0 to lie at the half-width at half-maximum (HWHM) of the central Airy peak or the half-width at $1/e^2$ of maximum (HW($1/e^2$)M) of the central peak, this later convention being justified by the fact that although the focal plane fluence profile (equation 3) is not Gaussian, the central peak is reasonably well approximated by a Gaussian, as Figure 2 illustrates. A disk of radius equal to the HWHM of the central peak has angular radius $w_0 = 0.257 \lambda / a$ and contains 47% of the energy passing through the aperture, while a disk of radius equal to the HW($1/e^2$)M of the central peak has $w_0 = 0.411 \lambda / a$ and contains 77% of the energy.

4. Numerical Example

To better illustrate the extent to which the numerical value of a reported fluence depends on particular convention used, we consider an aperture 10 cm in diameter admitting $1 \mu\text{J}$ of radiation at 532 nm. For an $f/5$ -, an $f/2.4$ -, and an $f/1.2$ -system, we compute the following:

- $F(0)$, the fluence at the geometrical focal point;

- $\bar{F}(w_0 = 0.610 \lambda / a)$, the average fluence over the central disk of the Airy pattern; and
- $\bar{F}(w_0 = 0.411 \lambda / a)$, the average fluence over a disk of radius equal to the $\text{HW}(1/e^2)M$ of the central Airy peak.

The results are summarized in Table 1.

Table 1. Fluence values for a 10-cm aperture admitting 1 μJ of energy at $\lambda = 532 \text{ nm}$.

Fluences (J/cm^2)	$f/5$	$f/2.4$	$f/1.2$
Fluence at geometric focal point	11.1	48.2	193
Average fluence of central Airy disk	2.53	11.0	44.0
Average fluence over a disk of radius the $\text{HW}(1/e^2)M$ of the central airy peak	5.10	22.1	88.5

5. Practical Formulae

We conclude with a compilation of practical formulae assembled for the benefit of workers measuring damage threshold fluences for focal plane elements. The formulae for average fluence are obtained by applying the general results equations 9 and 10 to particular choices for a “standard disk” discussed at the end of section 3. As before, E represents the total energy passing through the aperture, $(f/\#) = l/2a$ is the f -number of the system, and ρ_0 is the radius of the standard disk in question.

- From equation 4, the peak fluence, i.e., the fluence at the geometric focal point:

$$F(0) = 0.785 \frac{E}{\lambda^2 (f/\#)^2}.$$

- Average fluence over the central disk of the Airy pattern:

$$\bar{F}(\text{central disk}) = 0.267 \frac{E}{\rho_0^2} = 0.179 \frac{E}{\lambda^2 (f/\#)^2}.$$

- Average fluence over a disk of radius equal to the HWHM of the Airy peak:

$$\bar{F}(\text{HWHM disk}) = 0.151 \frac{E}{\rho_0^2} = 0.572 \frac{E}{\lambda^2 (f/\#)^2}.$$

- Average fluence over a disk of radius equal to the $\text{HW}(1/e^2)$ M of the Airy peak:

$$\bar{F}(\text{HW}(1/e^2)\text{M disk}) = 0.244 \frac{E}{\rho_0^2} = 0.361 \frac{E}{\lambda^2 (f/\#)^2}.$$