Universal Relations for Acceleration Wave Speeds in Nonlinear Viscoelastic Solids

by Michael J. Scheidler

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Universal Relations for Acceleration Wave Speeds in Nonlinear Viscoelastic Solids

Michael J. Scheidler
Weapons and Materials Research Directorate, ARL

# Universal Relations for Acceleration Wave Speeds in Nonlinear Viscoelastic Solids

For finite deformations of nonlinear viscoelastic solids, the speed of propagation of acceleration waves (i.e., ramp waves) generally depends not only on the current state of strain at the wave front but also on the prior strain history. Consequently, explicit formulas for the wave speed can be quite complicated. Simple formulas for the wave speed do exist for special classes of materials and/or special deformation histories, and in this regard we consider one-dimensional motions of viscoelastic solids governed by single integral laws. Some of the relations obtained are universal in the sense that they hold for all materials in a given class and do not explicitly involve the relaxation kernel function in the hereditary integral defining these materials.

### Subject Terms
- Acceleration waves
- Ramp waves
- Viscoelasticity
- Plate impact
UNIVERSAL RELATIONS FOR ACCELERATION WAVE SPEEDS IN NONLINEAR VISCOELASTIC SOLIDS

Mike Scheidler
US Army Research Laboratory, APG, Maryland 21005-5069

Abstract. For finite deformations of nonlinear viscoelastic solids, the speed of propagation of acceleration waves (i.e., ramp waves) generally depends not only on the current state of strain at the wave front but also on the prior strain history. Consequently, explicit formulas for the wave speed can be quite complicated. Simple formulas for the wave speed do exist for special classes of materials and/or special deformation histories, and in this regard we consider one-dimensional motions of viscoelastic solids governed by single integral laws. Some of the relations obtained are universal in the sense that they hold for all materials in a given class and do not explicitly involve the relaxation kernel function in the hereditary integral defining these materials.

INTRODUCTION

We consider the speed of propagation of acceleration waves in viscoelastic solids undergoing uniaxial strain, as would occur in a normal plate impact experiment prior to the arrival of lateral release waves. The front of a ramp wave is an example of an acceleration wave.\(^1\) Expansive (unloading) ramp waves can be generated by reflection of a shock wave from a free surface or a lower impedance material [1]. Compressive ramp waves can be generated by use of fused silica buffer plates or by graded density impactors [1, 3] and also by fast pulsed power techniques [3].

For nonlinear viscoelastic solids, the acceleration wave speed \(U\) is generally a nonlinear function of the current strain as well as the past strain history at the material point instantaneously situated on the wave front. In particular, for viscoelastic materials governed by single or multiple integral laws, the dependence of the wave speed on the strain history typically involves an explicit dependence on the relaxation kernel function(s) in the hereditary integral(s).\(^2\)

However, for special classes of viscoelastic solids and/or special strain histories, simple explicit formulas for the wave speed exist in terms of quantities which have a direct physical interpretation. An example of such a relation was given by Nunziato et al. [1] for an acceleration wave propagating into a deformed region in equilibrium in a finite linear viscoelastic solid.\(^3\) They showed that

\[
\rho U^2 = \frac{d\sigma}{d\varepsilon} + \frac{1}{1 - \frac{1}{2} \varepsilon_1} \frac{\sigma_1(\varepsilon_1) - \sigma_E(\varepsilon_1)}{\varepsilon_1},
\]

where \(\sigma_1\) and \(\sigma_E\) are the instantaneous and equilibrium elastic response functions, and \(\varepsilon_1\) is the equilibrium uniaxial strain ahead of the wave.\(^4\) This relation is universal in the sense that it holds for all materials in the indicated class and does not explicitly involve the relaxation kernel function \(G\) in the hereditary integral (17) defining the materials in this class.

The paper begins with a discussion of the general, one-dimensional, nonlinear, single integral law for viscoelastic response. This is followed by some general results on acceleration wave speeds in such materials. These results are used to derive relations analogous to (1) but for more general classes of nonlinear viscoelastic solids and non-equilibrium conditions ahead of the wave.

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\(^1\) More precisely, an acceleration wave is a propagating singular surface across which the stress, strain and particle velocity are continuous but their spatial gradients and time derivatives suffer jump discontinuities [1, 2].

\(^2\) See eq. (13) below for the general single integral case.

\(^3\) This class of nonlinear viscoelastic materials is defined by equations (17) and (20) below.

\(^4\) Precise definitions of all terms are given below.

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263
SINGLE INTEGRAL LAWS

Let \( \mathbf{F} \) denote the deformation gradient relative to a fixed, unstressed reference state, let \( J = \det \mathbf{F} \), and let \( \mathbf{T} \) denote the Cauchy stress tensor. Then \( \mathbf{S} = J \mathbf{T} (\mathbf{F}^{-1})^T \) and \( \mathbf{S} = J \mathbf{F}^{-1} \mathbf{T} (\mathbf{F}^{-1})^T \) are the 1st and 2nd Piola-Kirchhoff stress tensors. Introduce a Cartesian coordinate system with the 1-axis a symmetry axis of the material, and consider a time-dependent uniaxial strain along this axis. The normal component of stress, taken positive in compression, is

$$
\sigma = -T_{11} = -S_{11} = -\lambda_1 S_{11},
$$

where \( \lambda_1 = F_{11} \) is the (principal) stretch along the 1-axis. Let \( \varepsilon \) denote the nominal measure of uniaxial strain taken positive in compression: \( \varepsilon = 1 - \lambda_1 \).

We consider viscoelastic materials governed by a nonlinear single integral law, the most general one-dimensional form of which is

$$
\sigma(t) = \sigma_0(t) + \int_0^t \mathcal{G}(t,s) \varepsilon(t-s) ds.
$$

Here \( \mathcal{G} \) denotes the partial derivative of \( \mathcal{G} \) with respect to its third (or temporal) argument \( s \):

$$
\mathcal{G}'(\varepsilon_1, \varepsilon_2, s) = \frac{\partial}{\partial s} \mathcal{G}(\varepsilon_1, \varepsilon_2, s).
$$

Fading memory requires that the relaxation kernel \( \mathcal{G}'(\varepsilon_1, \varepsilon_2, s) \) decay to zero sufficiently rapidly as \( s \to \infty \). There is some non-uniqueness in the functions \( \sigma_0 \) and \( \mathcal{G} \). This may be removed by the assumptions

$$
\mathcal{G}'(\varepsilon_1, 0, s) = 0 \quad \text{and} \quad \mathcal{G}'(\varepsilon_1, \varepsilon_2, 0) = \sigma_0(\varepsilon_1).
$$

Indeed, since \( \mathcal{G}' \) rather than \( \mathcal{G} \) itself appears in the constitutive relation \( \sigma(t) \), we are free to choose the initial value \( \mathcal{G}(\varepsilon_1, \varepsilon_2, 0) \). The choice \( \mathcal{G}(\varepsilon_1, \varepsilon_2, 0) = 0 \) simplifies the results below. By \( \mathcal{G}(\varepsilon_1, \varepsilon_2, 0) = 0 \), the upper limit \( \infty \) in \( \sigma(t) \) may be replaced with \( t \) whenever \( \varepsilon(t) = 0 \) for \( t < 0 \). Condition \( \mathcal{G}'(\varepsilon_1, \varepsilon_2, 0) = 0 \) implies that \( \sigma_0(\varepsilon_1) \) is the instantaneous elastic response function, i.e., \( \sigma(t) = \sigma_0(\varepsilon_1) \) for the jump strain history

$$
\varepsilon(t) = \begin{cases} 
\varepsilon_1, & \text{if } t = 0; \\
0, & \text{if } t < 0;
\end{cases}
$$

then \( \sigma_0(0) = 0 \). Also note that \( \mathcal{G}(\varepsilon_1, \varepsilon_2, 0) \) implies

$$
\mathcal{G}(\varepsilon_0, 0, s) = \mathcal{G}(\varepsilon_0, 0, 0) = \sigma_0(\varepsilon_0).
$$

Observe that \( \sigma(t) \) is the strain history experienced by a point which, at the instant \( t \), lies on the front of a shock wave propagating into an unstrained region. Thus the instantaneous elastic response function may be inferred from measurements of the stress jump across shocks in undeformed materials [1].

For any material with fading memory, let \( \sigma_R(\varepsilon_1, t) \) denote the stress at time \( t \geq 0 \) for the stress relaxation test

$$
\varepsilon(\tau) = \begin{cases} 
\varepsilon_1, & \text{if } \tau \geq 0; \\
0, & \text{if } \tau < 0.
\end{cases}
$$

Then \( \sigma_R \) is called the stress relaxation function (cf. [4]), and

$$
\sigma_R(\varepsilon_1, 0) = \sigma_0(\varepsilon_1), \quad \sigma_R(\varepsilon_1, \infty) = \sigma_e(\varepsilon_1),
$$

where \( \sigma_e(\varepsilon_1) \) is the equilibrium elastic response function. From (3)\textendash(5) it follows that for \( t \geq 0 \),

$$
\mathcal{G}(\varepsilon_1, \varepsilon_2, t) = \sigma_R(\varepsilon_1, t),
$$

and that \( \mathcal{G}(\varepsilon_1, \varepsilon_2, t) \) is the stress at time \( t > 0 \) for the strain history

$$
\varepsilon(\tau) = \begin{cases} 
\varepsilon_1, & \text{if } \tau = t; \\
\varepsilon_2, & \text{if } 0 \leq \tau < t; \\
0, & \text{if } \tau < 0.
\end{cases}
$$

The 1-D linear theory of viscoelasticity, namely

$$
\sigma(t) = G(0) \varepsilon(t) + \int_0^t G'(s) \varepsilon(t-s) ds,
$$

is the special case of (3) with \( \mathcal{G}(\varepsilon_1, \varepsilon_2, s) = G(s) \varepsilon_2 \). Here \( G(s) = dG/ds; G(0) \) and \( G(\infty) \) are the instantaneous and equilibrium elastic moduli; \( \sigma_i(\varepsilon(t)) = G(0) \varepsilon(t) \) is the instantaneous elastic response; and \( \sigma_e(\varepsilon_1, t) \) is the stress response functional with respect to the current strain \( \varepsilon(t) \), holding

ACCELERATION WAVE SPEED

Let \( \rho_0 \) and \( \rho \) be the densities in the undeformed and deformed states. Let \( U(t) \) denote the referential or Lagrangean acceleration wave speed (measured with respect to distance in the undeformed reference state). For a general class of materials with fading memory, Coleman et al. [2] showed that \( \rho \frac{d^2U}{dt^2}(t) \) is given by the derivative of the stress response functional with respect to the current strain \( \varepsilon(t) \), holding
the past strain history fixed. When applied to the single integral law \((3)\), this yields the formula
\[
\rho_0 U^2(t) = \sigma_{\tau}^i(\epsilon(t)) + \int_0^\infty \partial_t G'(\epsilon(t), \epsilon(t-s), s) \, ds,
\]
(13)
where \(\sigma_{\tau}^i(\epsilon) = \frac{d}{d\epsilon} \sigma_\tau(\epsilon)\) and \(\partial_t G'\) denotes the partial derivative of \(G'\) with respect to its first argument. The Eulerian wave speed (measured with respect to distance in the deformed state) is given by \(U = \lambda_1 U_s\). And since \(\rho_0 = \lambda_1 \rho\), we also have \(\rho U^2 = \lambda_1 \rho_0 U^2\).

For the linear theory \((12)\), we see that \((13)\) reduces to \(\rho_0 U^2 = G(0)\), and hence we recover the well-known result that the wave speed is a constant, independent of the deformation ahead of the wave. However, for the nonlinear theory, \((13)\) implies that the acceleration wave speed \(U(t)\) is generally a complicated function not only of the current strain \(\epsilon(t)\) but also of the past strain history at the material point instantaneously situated on the wavefront.

If the material ahead of the wave was initially undeformed and subjected to a step in uniaxial strain of amount \(\epsilon_1\) at time zero (i.e., the stress relaxation test \((8)\)), then \((13)\) simplifies to
\[
\rho_0 U^2(t) = \partial_t G(\epsilon_1, \epsilon_1, t)
= \partial_t \sigma_\tau(\epsilon_1, t) - \partial_\tau G(\epsilon_1, \epsilon_1, t)
\]
(14)
for \(t > 0\), where \((14)\) follows from \((10)\).

More generally, for the strain history \((11)\) ahead of the wave, the acceleration wave speed at the instant \(t\) is given by
\[
\rho_0 U^2(t) = \partial_t G(\epsilon_1, \epsilon_1, t).
\]
(15)
Actually, this statement requires some qualification since \(\epsilon\) is not continuous at the instant \(t\). Let \(t > 0\) be fixed, and consider the strain history
\[
\epsilon(t) = \begin{cases} 
\dot{\epsilon}(\tau), & \text{if } \tau \geq t; \\
\epsilon_2, & \text{if } 0 \leq \tau < t; \\
0, & \text{if } \tau < 0; 
\end{cases}
\]
(16)
where \(\dot{\epsilon}(\tau)\) is any continuous function of \(\tau\) such that \(\dot{\epsilon}(t) = \epsilon_1\). Let \(U(\tau)\) denote the acceleration wave speed at time \(\tau > t\) for the strain history \((16)\) ahead of the wave. Then on taking the limit as \(\tau\) approaches \(t\) from above, we obtain \((15)\). That is, \((15)\) gives the acceleration wave speed in the state immediately following the second strain jump (at time \(t\)) for the strain history \((11)\).

**FINITE LINEAR VISCOELASTICITY**

Now we consider single integral laws of the form
\[
\sigma(t) = \sigma_\tau(\epsilon(t)) + \int_0^\infty G'(\epsilon(t), s) \cdot f(\epsilon(t-s)) \, ds,
\]
(17)
where \(G'(\epsilon_1, s) = \frac{d}{d\epsilon} G(\epsilon_1, s)\). This is the special case of \((3)\) with
\[
G'(\epsilon_1, \epsilon_2, s) = G'(\epsilon_1, s) \cdot f(\epsilon_2).
\]
(18)
To satisfy \((5)\) we require that
\[
G(\epsilon_1, \epsilon_2, s) = f(\epsilon_2) \left[ G(\epsilon_1, s) - G(\epsilon_1, 0) \right] + \sigma_\tau(\epsilon_1).
\]
(19)
Here \(f\) is interpreted as a strain measure, so that \(f(0) = 0\) and \(f'(0) = 1\), in which case \((5)\) is satisfied. Nunziato et al.\([1]\) considered the special case of \((17)\) with \(f\) given by
\[
f(\epsilon) = \epsilon - \frac{1}{2} \epsilon^2.
\]
(20)
This case arises from the 3-D single integral law
\[
S(t) = \mathcal{F}_f(E(t)) + \int_0^\infty G'(E(t), s) \left[ \mathbf{E}(t-s) \right] \, ds,
\]
(21)
where \(E = \frac{1}{2} \left( F^T F - I \right)\) is the Green strain tensor and \(G'(E(t), s)\) is a fourth order tensor. This class of viscoelastic materials was termed *finite linear* by Coleman and Noll\([5]\). The quadratic term in \((20)\) results from conversion to the nominal strain measure \(\epsilon\). Replacing \(E(t-s)\) by other finite measures of past strain results in a different class of materials and in particular a different choice for \(f\) in \((17)\).

On setting \(\epsilon_2 = \epsilon_1\) in \((19)\) and using \((10)\), we see that
\[
G(\epsilon_1, t) - G(\epsilon_1, 0) = \frac{\sigma_\tau(\epsilon_1, t) - \sigma_\tau(\epsilon_1)}{f(\epsilon_1)}.
\]
(22)
Now consider an acceleration wave in a material governed by the integral law \((17)\), with the material ahead of the wave undergoing the stress relaxation test \((8)\). Then from \((14)\), \((19)\) and \((22)\), the wave speed at time \(t > 0\) is given by
\[
\rho_0 U^2(t) = \partial_t \sigma_\tau(\epsilon_1, t) + \frac{f'(\epsilon_1)}{f(\epsilon_1)} \left[ \sigma_\tau(\epsilon_1) - \sigma_\tau(\epsilon_1, t) \right].
\]
(23)
Note that under the given assumptions, $\sigma_p(\epsilon_1, t)$ is the stress at the wave front. On taking the limit as $t \to \infty$ and using (9)$_2$, we obtain the speed of a wave propagating into a region which has been in equilibrium for all time at strain $\epsilon_1$ and stress $\sigma_p(\epsilon_1)$:

$$\rho_0 U^2 = \frac{d \sigma_p}{d \epsilon_1} + f'(\epsilon_1) [\sigma_0(\epsilon_1) - \sigma_p(\epsilon_1)].$$

(24)

For the special case where $f$ is given by (20), this reduces to the formula (1) of Nunziato et al. [1, §21]. They used this to calculate the speed of expansive acceleration waves propagating into deformed regions in equilibrium, the deformation having been induced by the passage of a steady shock wave. The strain history resulting from the passage of a steady shock is only approximately given by (8), but due to fading memory this approximation leads to small errors.

**PIPKIN-ROGERS MATERIALS**

Next we consider single integral laws of the form

$$\sigma(t) = \sigma_0(\epsilon(t)) + h(\epsilon(t)) \int_0^t G'(\epsilon(t-s), s) \, ds,$$

(25)

where $G'(\epsilon_2, s) = \frac{d}{ds} G(\epsilon_2, s)$, $G(0, s) = 0$, and $h(0) = 1$. This is the special case of (3) with

$$G'(\epsilon_1, \epsilon_2, s) = h(\epsilon_1) \cdot G'(\epsilon_2, s).$$

(26)

Condition (5)$_1$ is satisfied, and (5)$_2$ holds if we take

$$G(\epsilon_1, \epsilon_2, s) = h(\epsilon_1) \left[ G(\epsilon_2, s) - G(\epsilon_2, 0) \right] + \sigma_0(\epsilon_1).$$

(27)

Pipkin and Rogers [4] considered the 3-D single integral law

$$S(t) = G(E(t), 0) + \int_0^t G'(E(t-s), s) \, ds,$$

(28)

where $G(0, s) = 0$, so that the first term on the right represents the instantaneous elastic response. For the 1-D case considered here, (28) reduces to (25) with $h(\epsilon) = 1 - \epsilon = \lambda_1$; this term is a consequence of conversion from the second to the first Piola-Kirchhoff stress measure (see (2)). Replacement of $S$ in (28) with other (Lagrangean) stress tensors would result in different functional forms for $h$ in (25).

Now consider an acceleration wave in a material governed by the integral law (25), with no restrictions on the (uniaxial) strain history ahead of the wave. From (13) and (26), we see that $\rho_0 U^2(t)$ is given by $\sigma_0(\epsilon(t)) + h(\epsilon(t)) \int_0^t G'(\epsilon(t-s), s) \, ds$. Then on solving (25) for this integral and substituting the result into the above expression, we obtain

$$\rho_0 U^2(t) = \frac{d \sigma_0}{d \epsilon_1} + \frac{h'(\epsilon(t))}{h(\epsilon(t))} \left[ \sigma_0(\epsilon(t)) - \sigma(t) \right].$$

(29)

with $\epsilon(t)$ and $\sigma(t)$ the strain and stress at the wave front. When $h(\epsilon) = 1 - \epsilon$, the term $\frac{h'(\epsilon)}{h(\epsilon)}$ reduces to $1/(1 - \epsilon) = 1/\lambda_1$. For a stress relaxation test (8) ahead of the wave and $t > 0$, (29) reduces to

$$\rho_0 U^2(t) = \frac{d \sigma_0}{d \epsilon_1} - \frac{h'(\epsilon_1)}{h(\epsilon_1)} \left[ \sigma_0(\epsilon_1) - \sigma_0(\epsilon_1, t) \right].$$

(30)

Finally, we note that (29) and (30) remain valid if (25) is replaced by the more general relation

$$\sigma(t) = \sigma_0(\epsilon(t)) + h(\epsilon(t)) \cdot \psi(\epsilon(t-s)), \quad s > 0,$$

(31)

given appropriate restrictions on the functional $\psi$. In particular, it is assumed that $\psi$ does not depend on the current strain $\epsilon(t)$. This includes the case where $\psi$ is given by multiple hereditary integrals of the type considered by Green and Rivlin [6].

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