NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturer’s or trade names does not constitute an official endorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.
Color Controllable Polarization Entanglement Generation in Optical Fiber at Telecommunication Wavelengths

by Sanjit Karmakar
Department of Physics, University of Maryland, Baltimore County, Baltimore, MD 21250

and

Ronald E Meyers
Computational and Information Sciences Directorate, ARL

A reprint from Optical Society of America, 10 Aug 2015;23(16).

Approved for public release; distribution is unlimited.
This article proposes a polarized entangled photon source in optical fiber with low Raman noise that features the controllable generation of specified signal and idler wavelengths (colors) by varying the pump power. The novel two-color source can provide needed telecom entangled photon wavelengths for applications in quantum communications, quantum computing, and quantum imaging.
Color controllable polarization
entanglement generation in optical fiber
at telecommunication wavelengths

Sanjit Karmakar and Ronald E. Meyers

1Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250, USA
2U.S. Army Research Laboratory, Adelphi, Maryland 20783, USA

Abstract: This article proposes a polarized entangled photon source in optical fiber with low Raman noise that features the controllable generation of specific signal and idler wavelengths (colors) by varying the pump power. The novel two-color source can provide needed telecom entangled photon wavelengths for applications in quantum communications, quantum computing, and quantum imaging.

OCIS codes: (270.5290) Photon statistics; (110.0110) Imaging systems.

References and links
Entanglement, a quantum mechanical phenomenon in which the interacting states of two or more quantum systems cannot be described independently, is vital to a wide range of new technologies such as quantum imaging [1–6], quantum computing [7, 8] and teleportation [9, 10].

Despite its importance, robust entangled photon generation and control with specific needed wavelengths (colors) is difficult to achieve. Two common sources of entangled light are based on non-linear optical interaction of parametric downconversion (PDC) and four-wave mixing (FWM) in both spontaneous (SPDC, SFWM) and stimulated regimes. The SPDC process is commonly used to generate entangled photon pairs [11–13]. In addition, waveguide SPDC was developed to increase the rate of entangled photon pair generation, i.e. the brightness [14, 15]. Nevertheless, SPDC in bulk crystals and waveguides is limited by the spectral width of the pump source. Yet, this limitation could be overcome by using quasi-phase matching, birefringence, and self phase or cross phase modulation features of the material.

1. Introduction

Entanglement, a quantum mechanical phenomenon in which the interacting states of two or more quantum systems cannot be described independently, is vital to a wide range of new technologies such as quantum imaging [1–6], quantum computing [7, 8] and teleportation [9, 10].

Despite its importance, robust entangled photon generation and control with specific needed wavelengths (colors) is difficult to achieve. Two common sources of entangled light are based on non-linear optical interaction of parametric downconversion (PDC) and four-wave mixing (FWM) in both spontaneous (SPDC, SFWM) and stimulated regimes. The SPDC process is commonly used to generate entangled photon pairs [11–13]. In addition, waveguide SPDC was developed to increase the rate of entangled photon pair generation, i.e. the brightness [14, 15]. Nevertheless, SPDC in bulk crystals and waveguides is limited by the spectral width of the pump source. Yet, this limitation could be overcome by using quasi-phase matching, birefringence, and self phase or cross phase modulation features of the material.
Because optical fiber is well suited for the transmission of quantum information, the progress of photon pair generation in optical fibers by FWM has gained a great deal of attention [16–22]. However, FWM presents a high probability of Raman noise contamination of the idler photon of the generated pair specifically at low pump power. While Raman noise can be avoided in the case of liquid core fibers, these materials have not yet been applied to the generation of telecom wavelength entangled photons [23]. Fortunately, optical fibers also have a 3rd-order nonlinearity feature for self phase and cross phase modulations. These modulation effects can be used to generate two specified colors (wavelengths) of entangled photon pairs at a high rate.

We propose a polarization entanglement source in optical fiber at telecommunication wavelengths which features the controllable generation of specific colors (wavelengths) of entangled photon pairs. In particular, this proposed experiment builds on and can be distinguished from earlier work by Fang [18], Chen [19], Li [20], and Takesue [21] by our use of increased peak pulse power to tune the phase matching conditions in the optical fiber. Our source can generate entangled photon pairs at wavelengths separated from the Raman contamination of the pump.

2. Experimental setup

The proposed experimental setup for the color controllable polarized entanglement source will probabilistically generate polarization entangled photons according to the quantum wavefunction

$$|\Psi_p\rangle = \frac{1}{\sqrt{2}} \left[ |H\rangle_s |H\rangle_i + e^{2i\phi_p} |V\rangle_s |V\rangle_i \right].$$

(1)

Here, the relative phase between the two orthogonally polarized pump pulses is given by $\phi_p$; the horizontally and vertically polarized pulses are $H$ and $V$. As usual, $s$ represents the signal and $i$ is the idler. Appropriate settings of $\phi_p$ and a half-wave plate can be used to produce all four Bell states in the polarization degree of freedom [20].

Figure 1 shows a schematic of the proposed experimental setup to generate signal and idler photon pairs in an optical fiber. A pump pulse is passed to a Sagnac loop through a 50-50 beam splitter. BS is a 50-50 beam splitter and DM is a dichoric mirror which transmits idler photons and reflects signal photons. PM is polarization maintaining and PBS is a polarized beam splitter. LLM is a laser line mirror which is used to remove the pump. V- and H- represent vertical and horizontal polarization, respectively. Also, PZT is a piezoelectric transducer driven translation stage, which controls the delay between V-polarized and H-polarized pulses. Later, a PM-delay compensator is used to compensate the delay to overlap H- and V- polarized signal and idler photons. Two quarter wave plates (QWP) are also used to avoid reflection back into the laser. PC is a polarization controller.
splitter. The Sagnac loop consists of an optical fiber with a polarization controller which is used to maintain polarization of the signal and idler to be the same as that of the pump. The FWM process occurs in the Sagnac loop to generate signal-idler photon pairs. A photon pair is created in each of the clockwise and counter-clockwise propagating Sagnac loop paths with equal probability producing superposition and interference [18].

In the proposed experiment, a delay of a few picoseconds (ps) is introduced between two orthogonally polarized pump pulses by adjusting the mirror mounted on a piezoelectric-transducer (PZT) driven translation stage. This precisely adjusts the relative delay, i.e. phase difference, between the two orthogonally polarized pump pulses. When the H and V temporal indistinguishability is restored by the polarized-maintained (PM) delay compensator the generated state is given by Eq. (1) where \( \phi_p \) is the relative phase between the states. After the delay compensator, the generated signal and idler photons from the Sagnac loop are passed through a dichroic mirror to separate the signal and idler photons.

In a FWM process, two pump photons of angular frequencies \( \omega_p \) are converted to two daughter photons at angular frequencies \( \omega_s \) and \( \omega_i \) in a third-order nonlinear optical material, such as optical fiber. This third-order nonlinearity generates two side bands, i.e. signal and idler beams. During this process energy is conserved and the phase-mismatch \( \Delta k \) is 0. Conditions for the co-polarized, i.e. pump, signal and idler have the same polarization, case are [17, 24],

\[
\begin{align*}
2\omega_p &= \omega_s + \omega_i \\
\Delta k &= 2 \frac{\omega_p}{c} n(\omega_p) - \frac{\omega_s}{c} n(\omega_s) - \frac{\omega_i}{c} n(\omega_i) - \gamma P_p
\end{align*}
\]

where \( s \) and \( i \) refer to the signal and idler photons respectively, \( \gamma \) is the non-linear parameter of the fiber, \( P_p \) is the pump peak power and \( n(\omega) \) is the refractive index at angular frequency \( \omega \). At high pump peak power, the self- or cross-phase modulation term (a self/cross-phase modulation contribution comes from the two pumps each with peak power \( \frac{P_p^2}{2} \)), \( \gamma P_p \), can control the phase-matching where \( \gamma = \frac{3\pi|\chi^{(3)}|}{2\lambda_p A_{eff} n^2 n_0 c} = \frac{2\pi n_2}{\lambda_p A_{eff}} \), \( n_2 \) is nonlinear index of the material and \( \lambda_p \) is center wavelength of the pump. The terms \( n, A_{eff} \) and \( \chi^{(3)} \) are average refractive index, effective mode area and third-order susceptibility of the nonlinear fiber medium respectively. We also have the following wavelength and bandwidth relations

\[
\frac{1}{\lambda_i} + \frac{1}{\lambda_s} = \frac{2}{\lambda_p},
\]

\[
\sigma_w = \frac{2\pi c}{(\lambda_w)^2} \Delta \lambda_w,
\]

where \( w = p, i, s \) and

\[
\frac{\sigma_u}{\sigma_p} = \left( \frac{\lambda_p}{\lambda_u} \right)^2 \left( \frac{\Delta \lambda_u}{\Delta \lambda_p} \right).
\]

The bandwidth of the pump is \( \sigma_p \) and \( \sigma_u \) are the bandwidths of the idler and signal where \( u = i, s \). The terms \( \lambda_i, \lambda_s \) and \( \Delta \lambda_i, \Delta \lambda_s \) are determined by the phase matching condition \( \Delta k = 0 \).

3. Analysis of entangled photons generated, color controllability, and noise coincidences

3.1. Raman noise

Now a laser with a 1560 nm emission wavelength, average power of 100 mW, repetition rate of 73.4 MHz and a pulse duration of 150 fs is considered as the pump to generate signal and idler beams in an optical fiber. Here pump, signal and idler are considered as being in an...
extraordinary polarized state. Using Eq. (2), Eq. (3), and Sellmeier’s equations [25], Fig. 2 shows the phase matching for the 1560 nm pump having 100 mW average power. From Fig. 2, we can see that phase matching occurs at a signal wavelength of 1537 nm at an average power of 100 mW, where the corresponding idler wavelength would be 1583.5 nm. Similarly, the occurrence of phase matching for other average pump powers of 20 mW, 200 mW and 300 mW are also shown in Fig. 2 and illustrate how phase matching would shift with average pump power. Figure 3 displays the variation of the signal and idler wavelengths with the pump power. Again, we can see that as the pump power increases, both wavelengths of signal and idler photons move away from that of the pump photons. At low power, the wavelength of the signal and idler photons are close to that of the pump.

For a 1560 nm pump, stimulated Raman scattering would bring about a red-shift of approximately 100 nm [26]. As a result, the generated idler photons would have a high probability of Raman contamination. In general, for stimulated Raman contamination to occur a minimum pump power called the threshold pump power is needed. The Raman threshold pump can be defined as [27] \( P_{th} \simeq \frac{16A_{eff} }{g_R L_{eff}} \), where \( A_{eff} = 76.94\mu m^2 \) is the mode area of the fiber and \( g_R = 6.34 \times 10^{-12} \) is the maximum Raman gain in \( cm/W \) [26]. At 1560 nm, \( L_{eff} = 22 \) km. Thus, the Raman threshold would be 882 mW. In this case, the fiber length \( L \) is 20 cm, which is much smaller than \( L_{eff} = 22 \) km and the threshold power can be rewritten as \( P_{th} \simeq \frac{16A_{eff} }{g_R L} = 97 kW \).

In addition, the 1560 nm pump has a 150 fs pulse duration, a 73.4 MHz repetition rate and a 100 mW average power to generate photon pairs from each pulse. These specifications of the pump equate to a peak power of 9.08 kW, which is much smaller than the Raman threshold.
power. Hence, in this implementation, there would be negligible probability of contamination due to stimulated Raman scattering.

Below the threshold spontaneous Raman photons are still generated. The number of Raman (either Stokes or anti-Stokes) photons per pump pulse is calculated as [28, 29]

$$R_c = P_p L N_{up} \frac{|g_R| \sigma_u}{\sigma_p}$$  \hspace{1cm} (7)

where

$$N_{up} = \begin{cases} n(\Omega_{up}) & \text{when } \Omega_{up} > 0 \\ n(\Omega_{up}) + 1 & \text{when } \Omega_{up} < 0 \end{cases}$$

and $\Omega_{up} = \Omega_a - \Omega_p$, $k_B$ is the Boltzmann constant, $T$ is a temperature, $g_R$ is the Raman gain and $u = s, i$. Basically, the Stokes photons contaminates the idler photon where the signal is contaminated by anti-Stokes photons. The generation of Stokes and anti-Stokes photons are shown in Fig. 4. Only at low power is the noise due to Raman photons comparable to the photon pairs. As the power increases, the noise due to Raman photons is negligible in comparison to the signal and idler photons.

Fig. 3. Wavelength of the signal and idler photons as a function of average pump power.
3.2. Coincidence generation

The quantum state of signal and idler pair generated via four-wave mixing at the output of fiber [16] can be calculated through a perturbation expansion [30] as

\[ |\psi\rangle = |0\rangle + gL \sum_{k_s,k_i} F(k_s,k_i) a^\dagger_{k_s} a^\dagger_{k_i} |0\rangle + (gL)^2 \sum_{k_s,k_i,k'_s,k'_i} F(k_s,k_i,k'_s,k'_i) a^\dagger_{k_s} a^\dagger_{k_i} a^\dagger_{k'_s} a^\dagger_{k'_i} |0\rangle + \ldots \] (8)

where

\[ F(k_s,k_i) = \frac{1}{L} \int_{-L}^{0} dz \exp \left[ i\Delta k z - \frac{(\nu_s + \nu_i)^2}{4\sigma_p^2} \right] \] (9)

is the two-photon spectral amplitude [31] and \( F(k_s,k_i,k'_s,k'_i) \) is the four-photon spectral amplitude. Higher order terms involving multi-photon states are relatively small and therefore can usually be neglected [16]. Here \( \Delta k \) is phase-mismatching, \( g = \frac{\alpha_e n^2 A_{\text{eff}}(1)}{\alpha_p \nu_p^2 \sigma_p^2} \), the group velocity dispersion (GVD) in the fiber is negligible, \( \alpha_e \) is a constant determined by experimental conditions, the wavevector \( k = \frac{n(\omega)}{\omega} c \), \( V_Q \) is the quantization volume, \( \sigma_p \) is the bandwidth of the pump, \( \omega_s = \Omega_s + \nu_s \), \( \omega_i = \Omega_i + \nu_i \), and \( \omega_p = \Omega_p + \nu_p \), where \( \Omega \) and \( \nu \) are the center angular frequencies and their corresponding detuning frequencies, respectively.

Regarding the higher order terms of Eq. (8), the value of \( g \) in ordinary silicon fiber is \( |g| = 3.468 \times 10^{-6} \) per unit length (in m) of fiber for a pump with average power of 300 mW. In 20 cm fiber, the value \( |g|L \) is 6.936 \( \times 10^{-7} \). The values of the higher order perturbation coefficients in the quantum state shown in Eq. (8) are proportional to \( (|g|L)^m \) where \( m = 1, 2, 3, \ldots \) represent \( 1^{st}, 2^{nd}, 3^{rd}, \) and \( m^{th} \) order perturbations. Due to the small value of \( |g|L \), higher orders powers can be safely neglected.

The signal photon counting probability of a photon in each pulse is given by [16]

\[ S_s = \int_0^\infty dT \langle \psi | E_s^{(-)}(T) E_s^{(+)}(T) | \psi \rangle \] (10)

where the electric field operator is

\[ E_s^{(+)} = \sum_{k_s} \sqrt{\frac{c A_{\text{eff}}}{4V_Q}} a_{k_s} e^{-i\omega_s t} e^{-\left(\frac{\omega_s - \Omega_s}{2\sigma_s}\right)^2} \] (11)

where \( \sigma_s \) is the bandwidth of the signal pulse, \( a_{k_s} \) is an annihilation operator of a photon with momentum \( k_s \). Now the integrand can be written as

\[ \langle \psi | E_s^{(-)}(T) E_s^{(+)}(T) | \psi \rangle = \frac{c A_{\text{eff}}}{4V_Q} (gL)^2 \sum_{k_s} \sum_{k_i} F(k_s,k_i) e^{-i\omega_s t} e^{-\left(\frac{\omega_s - \Omega_s}{2\sigma_s}\right)^2} \] (12)

After simplification Eq. (10) at perfect phase-matching condition, \( \Delta k = 0 \), becomes

\[ S_s = A (\gamma P_p L)^2 \frac{\sigma_s}{\sigma_p} \] (13)

where \( A = \frac{\alpha^2 \pi n n_{\text{eff}}^3}{18N^2 V_Q^3} \) and \( n \) is the average refractive index of fiber material. Similarly, the idler photon counting probability of a photon in each pulse can be calculated as,

\[ I_i = A (\gamma P_p L)^2 \frac{\sigma_i}{\sigma_p} \] (14)
where $\sigma_i$ is the bandwidth of generated idler pulse.

Here, the nonlinear coefficient of the refractive index is $n_2 = 3.4 \times 10^{-20} \text{m}^2/\text{W}$. For 1560 nm optical fiber, $n = 1.5$, $A_{\text{eff}} = 76.94 \mu\text{m}^2$ and the value of $\gamma$ becomes $1.7904 \times 10^{-9} \mu\text{m}^{-1}\text{W}^{-1}$. The parameters $\alpha_e$ and $V_Q$ are given by $\alpha_e = 0.237$ and $V_Q = 1.6 \times 10^{-16} \text{m}^3$ [16]. Equations (13) and (14) show that the number of signal and idler photons generated from a pulse increases with the pump power. This result is shown in Fig. 4.

![Figure 4. Number of generated signal, idler and Raman (Stokes & Anti-Stokes at T=20 K) photons per pulse as a function of the average power of pump. The inset highlights the difference of signal-idler and Raman photons at low power. The change in the accidental Raman single photon rate is due to a change in the Raman gain as a function of the Raman shift off of the pump [29].](image)

The probability of a coincidence from each pulse and assuming no loss can be written as [16]:

$$CC = \int_{0}^{\infty} dt_1 \int_{0}^{\infty} dt_2 \langle \psi | E_1^{(-)} E_2^{(-)} E_1^{(+)} E_2^{(+)} | \psi \rangle$$  \hspace{1cm} (15)

where the fields at the output fiber measured by the detector 1 and detector 2 are given by

$$E_1^{(+)} = \sum_{k_1} \sqrt{\frac{c A_{\text{eff}}}{4 V_Q}} a_{k_1} e^{-i \omega_{k_1} t_1} e^{-\frac{(\omega_{k_1} - \Omega_s)^2}{2 \sigma_s^2}}$$  \hspace{1cm} (16)

and

$$E_2^{(+)} = \sum_{k_2} \sqrt{\frac{c A_{\text{eff}}}{4 V_Q}} a_{k_2} e^{-i \omega_{k_2} t_2} e^{-\frac{(\omega_{k_2} - \Omega_i)^2}{2 \sigma_i^2}}$$  \hspace{1cm} (17)
Here, $t_m = T_m - l_m/c$ is the time at the output tip of the fiber and $l_m$ is the optical path length from output tip of the fiber to the detectors, $m = 1, 2$ and $T_m$ is the registration time at $m^{th}$ detector. Now the integrand in Eq. (15) can be rewritten as

$$\langle \psi | E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} | \psi \rangle = |\langle 0 | E_2^{(+)} E_1^{(+)} | \psi \rangle|^2$$

(18)

Since $\langle 0 | E_1^{(+)} E_2^{(+)} | \psi \rangle \approx \langle 0 | a_{k_2}^+ a_{k_1}^+ a_{k_1} a_{k_2} | 0 \rangle$, this (integrand) term only exists at $k_1 = k_2$ and $k_2 = k_i$ (see Ref. [16], p. 5). Using the two-photon state in Eq. (8) and the fields defined in Eqs. (16) and (17), the two-photon amplitude, Eq. (18) can be written as,

$$\langle 0 | E_2^{(+)} E_1^{(+)} | \psi \rangle = \frac{cA_{eff}}{4V_0} sL \sum_{k_i,k_j} F(k_s,k_i)e^{-i(\alpha_0 t_1 + \alpha_0 t_2)} \times e^{-\frac{v_s^2 + v_i^2}{2\sigma_s^2}}$$

(19)

where $\sigma_s = \sigma_i$ is considered for this case. After simplification, the probability of a coincidence from each pulse (without any Raman noise), i.e. Eq. (15) at the condition of $\Delta k = 0$ becomes

$$CC = B(\gamma P_p L)^2 \frac{\sigma_s^2}{\sigma_p \sqrt{\sigma_s^2 + \sigma_i^2}} = B(\gamma P_p L)^2 \frac{1}{\sigma_p \sqrt{\left(\frac{\sigma_s}{\sigma_i}\right)^2 + 1}}$$

(20)

where $B = \frac{\sigma_s^2 \pi^2 A_{eff}^2}{144V_0^2 L^3}$. Note that in Eq. (20) the coincidence counts scale quadratically with the pump power through the term $(\gamma P_p L)^2$ [16].

For our proposed experiment the phase modulation term $\gamma P_p$ plays a major role in phase matching. This phase modulation also causes pulse broadening which means all pump, signal, and idler bandwidths would be increasing according to their wavelengths. The ratios of the bandwidths of idler or signal and pump remain the same as that without pulse broadening. Since both single counts and coincidences depend on the ratio of the bandwidths of idler or signal and pump, singles counts and coincidences would be largely unaffected by pulse broadening. Furthermore, because our proposed experiment would use a short (20 cm) non-polarization maintaining fiber to generate entangled photons the asymmetric spectral reshaping of the type measured in the experiment by Fang et al. [18] would be avoided. We anticipate insignificant changes, caused by spectral reshaping, to the quality of entanglement for our proposal.

The Raman noise also contaminates the coincidence counts, where the observed coincidence counts $(CC_{obs})$ can be written as

$$CC_{obs} = CC + CC_{acc}$$

(21)

and the accidental coincidence $CC_{acc}$ can be written as $CC_{acc} = CC_{sSt} + CC_{sSt} + CC_{iAs}$. $CC_{sSt}$ are coincidences between Stokes and Anti-Stokes photons, $CC_{sSt}$ the coincidences between signal and Stokes photons, and $CC_{iAs}$ are coincidences between idler and anti-Stokes photons. At higher powers as shown in Fig. 5 the predicted coincidences and accidental coincidences may include contributions from higher order correlated or entangled photon states.

The dependence of Raman noise on temperature is given by Eq. (7). Cooling the Sagnac fiber loop components is one method that can be used to reduce the number of accidental coincidences caused by Raman noise to well below the true coincidence counts $(CC)$ (Fig. 5). There are several potential methods to cool the Sagnac fiber loop. Using a helium based cryostat can cool from $1^\circ K$ to $5^\circ K$ while hydrogen based or closed cycle cryostats can cool to $20^\circ K$. For example, Tittel et al. were successful in cooling fibers to $1^\circ K$ [32].
Fig. 5. Number of generated coincidences in each pulse is shown as a function of the average pump power. The inset represent a clear vision of the difference of true coincidences and accidental coincidences due to Raman scattering at low pump power. The change in the accidental Raman coincidence rate is due to a change in the Raman gain as a function of the Raman shift off the pump [29].

3.3. Single and coincident photon generation

It is well known that two of the reasons for the lack of one to one correspondence between singles and coincidences in the proposed FWM experiment are: (1) that the B over A coincidence and single photon generation coefficients are determined by fiber quantization properties and (2) $\sigma_p$, $\sigma_i$, and $\sigma_s$ which are the bandwidths of the pump, signal and idler respectively [16]. This is true even for ideal detectors.

The normalized second order correlation $g^{(2)}$ can be written as [28]

$$g^{(2)} = 1 + \rho_c,$$

where $\rho_c$ is the pair correlation for arbitrary polarization angles. We note that a $\rho_c$ violation of the Cauchy-Schwarz inequality is a sign of nonclassicality. In practice, a large $g^{(2)} \gg 2$ when evaluated by [28,33]

$$g^{(2)} = \frac{CC}{S_i I_c},$$

is an indicator not only of nonclassicality but also of entanglement. To gain insight into the dependence of $g^{(2)}$ on bandwidths we note that when $\sigma_s \simeq \sigma_i$

$$g^{(2)} \sim \frac{B}{A^2} \frac{1}{(\gamma P_p L)^2} \frac{1}{\sqrt{1 + \left(\frac{\sigma_i}{\sigma_p}\right)^2}},$$

Received 5 Mar 2015; revised 3 Jul 2015; accepted 3 Jul 2015; published 29 Jul 2015

© 2015 OSA
which depends explicitly on the ratio $\frac{\sigma_s}{\sigma_p}$. The nonclassicality, as indicated by large $g^{(2)}$, can be increased if the nonlinearity $\gamma$, the pump power $P_p$, or length $L$ can be reduced. In an idealized detection condition, with unit quantum efficiency of the detectors, no loss, $\sigma_z \simeq \sigma_i \simeq \sigma_p$, and no dark counts $g^{(2)} \simeq \frac{B}{A^2(\gamma P_p L)^2} \sqrt{\frac{1}{2}}$. Under other conditions, when the pump bandwidth $\sigma_p$ is much narrower than the idler bandwidth $\sigma_i$, i.e. $\sigma_p \ll \sigma_i$, $g^{(2)}$ approaches zero. When the pump bandwidth is much wider than the idler bandwidth, $\sigma_p \gg \sigma_i$, $g^{(2)}$ approaches a constant, $g^{(2)} \simeq \frac{B}{A^2(\gamma P_p L)^2}$.

Fig. 6. Normalized second order correlation $g^{(2)}$ as a function of the average pump power.

Quantum detection efficiency is the ability of the detector to annihilate one incoming photon into one electron. In case of unit quantum detection efficiency, each photon is annihilated into one electron at the detector. For a single photon measurement, this annihilation happens in one detector whereas this annihilation happens with two detectors simultaneously for a coincident photon pair measurement. The ratio of coincidence counts and single counts in the case where $\sigma_z \simeq \sigma_i$ can be written as

$$\frac{CC}{Sc} = \frac{B}{A} \frac{1}{\sqrt{1 + \left(\frac{\sigma_p}{\sigma_i}\right)^2}}$$

(25)

for Gaussian shapes of the pulsed pump, signal and idler fields. The value of counts ratio $\frac{CC}{Sc}$ explicitly depends on the quantity of $\frac{\sigma_p}{\sigma_i}$. The empirical parameters based on measurements in fiber with Gaussian fields lead to the value of $\frac{B}{A} \simeq 0.69$. In the limit of $\sigma_p \simeq \sigma_i$,

$$\frac{CC}{Sc} = \frac{B}{A} \frac{1}{\sqrt{2}}.$$  

(26)
From Eq. (25) at $\sigma_p \simeq \sigma_i$, the calculated value of $\frac{CC}{N_c}$ is 0.48 for all pump power values. Since $\frac{CC}{N_c}$ is the ratio of coincidence counts to singles counts each coincidence count is 0.48 times the number of singles counts in a pulse. This is very efficient for the FWM broadband process. For our process and the broadband formulation, the pump, signal, and idler fields are Gaussian distributed and for the wavelengths in question have approximately equal bandwidths. Mathematically the bandwidths are equivalent to filter bandwidths. The source pulses are not delta functions but have distributions in frequency. It appears that the drop in efficiency from one is the result of the Gaussian nature of the pulses and the values of the empirical coefficients in B and A. We note that in some FWM fiber based experiments this linearity may be absent due to the broadband nature of the pump field and the presence of filters because some of the correlated photon pairs are lost due to filtering and some uncorrelated photons are detected [16].

The normalized second order correlation, $g^{(2)}$, in the proposed experiment increases from a small value to approximately 6.5 as the average pump power decreases from 300 mW to 20 mW as shown in Fig. 6. Even higher $g^{(2)}$ are possible and feasible with lower average pump power and appropriate filtering. A relatively large $g^{(2)}$ can indicate not only nonclassicality but is also a signature of entanglement [28,34]. For our proposed experiment the pump bandwidth is 24 nm and the signal and idler wavelengths are chosen beyond 1560 ± 12 nm to avoid background due to the pump. At lower average pump powers the wavelength separation between the signal and idler becomes smaller. In general, to avoid pump photons from contaminating photons in the signal and idler wavelength bands the pump may be filtered to be narrower than the separation of the signal and idler wavelengths. For example, using a 10 nm filter an average pump power of 20 mW will generate signal and idler photons at 1550 nm and 1570 nm respectively. Then the signal and idler photons are beyond 1560 ± 5 nm. Furthermore, while Raman noise degrades the single or idler correlations, absorption may lower the accidental coincidences relative to entangled pairs which can increase the non-classicality [33]. By comparison Tapster et al. [35], for a broadband SPDC process calculated the two photon correlation $g^{(2)}$ to be equal to two at a narrow filter bandwidth and one at larger filter bandwidths.

Our proposed source entangled photon pair generation rate is 2.64 million pairs per second at 20 mW average pump power calculated as 0.036 pair per pulse at 73.4 MHz pulse rate. Notably, Fang et al. [18] measured a visible-near infrared pulsed entangled photon pair generation rate at specific polarizer settings of approximately 10000 per second from 5 mW of average pump power and Dong et al. [36] approximately 1000 entangled telecom wavelength photon pairs per second from a 3 mW continuous pump source.

4. Conclusions

In conclusion, our analysis indicates that polarization entangled photon pairs can be generated in an optical fiber by a FWM process. The key feature of this entangled photon source is the controllable generation of specific signal and idler colors (wavelengths). In this case, the color (wavelength) selections may be accomplished by varying the pump power. Another important quality of this source is its feature of low Raman noise photon pair generation. These kinds of color controllable entangled photon pairs would be useful in certain applications for quantum communications, quantum computing, and quantum imaging. We expect that this single spatial mode source will be useful for fundamental studies investigating real time quantum imaging of entanglement such as by extending the single wavelength experiment of Fickler et al. [37] to include the controllable generation of specific signal and idler wavelength entangled photons.

Acknowledgments

The authors thank K. Deacon, A. Tunick, and P. Hemmer for helpful discussions. This research was supported and funded by the U.S. Army Research Laboratory (ARL). S. Karmakar is a Post-Doctoral Fellow at ARL (W911NF-11-2-0074).