Data-Driven Analysis of Geometry and Topology Inspired Network Models and Metrics

S&T Campaign: Information Sciences
Networks and Communication

Terrence J Moore, (301) 394-1236
terrence.j.moore.civ@mail.mil

Research Objective

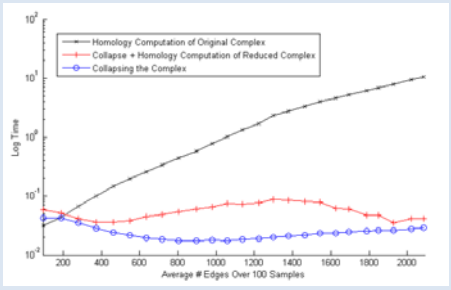
- **Geometric and topological methods applied to develop theory, algorithms and techniques for structural and behavioral classifications and decisions in dynamic or evolving networks**
  - Adapt novel mathematical concepts (e.g., curvature, homology) on network manifolds derived from real data for application to processes on networks
  - Use higher-dimensional representations of networks (e.g., simplicial complexes) to capture group interactions and relationships

Graph	$\overline{OR}$	$\overline{F}$	$\overline{JC}$	$\overline{gJC}$	(OR,F)		(OR,JC)		(OR,gJC)	
					r_p	τ	r_p	τ	r_p	τ
US power grid	-0.34	-3.7	-1.89	-0.78	0.48	0.41	0.40	0.23	**0.80**	**0.69**
EuroRoad	-0.33	-2.0	-1.97	-0.97	**0.76**	**0.69**	0.15	0.09	0.69	0.67
PGP network	-0.10	-33	-1.36	-0.14	0.13	0.08	0.73	0.53	**0.85**	**0.74**
p2p-Gnutella	-1.01	-31	-1.98	-1.16	-0.32	0.08	0.23	0.27	**0.86**	**0.58**
Email network	-0.41	-33	-1.72	-0.38	0.15	0.11	0.73	0.56	**0.81**	**0.69**
Hamsterster friendships	-0.34	-86	-1.87	-0.19	0.13	0.13	0.41	0.23	**0.58**	**0.42**
Human protein	-0.62	-27	-1.93	-0.79	0.35	0.34	0.31	0.10	**0.78**	**0.60**
Jazz musicians	0.27	-73	-0.92	0.32	0.09	0.05	0.91	0.79	**0.92**	**0.80**

Comparisons of Ollivier-Ricci (OR) curvature with Forman (F) curvature and two Jaccard curvatures (JC,gJC) on several network datasets using Pearson's r and Kendall's τ correlation coefficients

Challenges

- **Meaningful interpretation of discovered concepts for multidisciplinary network science**
- **Higher complexity is inherent to problem space due to combinatoric nature of beyond pair-wise interactions**



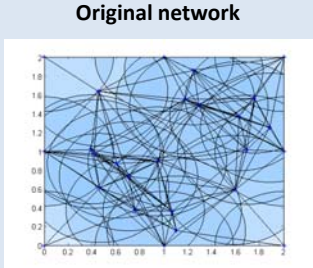
		Time complexity for computing Betti sequence		
dataset	Betti seq.	none	elementary	strong+elementary
Singapore chat/email	1,6,3,3,0,...	*	*	44s
Enron email	3,32,9,0,1,0,...	*	13.3s	1.2s

*** MATLAB out of memory error or crash**

Reduction of computational complexity to due to collapsing in random network and small chat/email datasets

ARL Facilities and Capabilities Available to Support Collaborative Research

- **Network Science Research Laboratory**
  - Integrated framework for experimentation on networks
- **Code for visualization and computation (MATLAB/Python)**
- **Results:**
  - Power law in facet (group) degree distribution
  - Jaccard-inspired approximation of Ollivier-Ricci curvature
  - Topological data analysis used to detect temporal network signatures and classify networks
  - Strong collapsing theory (e.g., preservation of minimal-cycle homology generators) applied to hole localization and sparse coverage in location-unaware sensor networks



Coverage redundancy in GPS- denied sensor network eliminated using coverage-preserving strong collapsing approach

Complementary Expertise/ Facilities/ Capabilities Sought in Collaboration

- **Expertise in advanced topology, geometry, or combinatorics theory with applications**
- **Currently exploring potential extensions of persistent homology and differential geometry approaches in social network dynamics**

Publications:

- Pal, Yu, Moore, Ramanathan, Bar-Noy, Swami, "An efficient alternative to Ollivier-Ricci curvature based on the Jaccard metric," arXiv 2017
- Pal, Moore, Ramanathan, Swami, "Comparative topological signatures of growing collaboration networks," CompleNet 2017
- Moore, "Greedy approaches to finding a sparse cover in a sensor network without location information," GlobalSIP 2016
- Wilkerson, Moore, Swami, Krim, "Simplifying the homology of networks via strong collapses," ICASSP 2013