Optimal Particle Size for the Cold Spray Process

by
Dennis Helfritch and Victor Champagne
U.S. Army Research Laboratory
Aberdeen Proving Ground, MD

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Modeling Efforts

Gasdynamic equations are used to calculate gas velocity and temperature within the nozzle and downstream of the nozzle exit.

The resulting particle velocity and temperature are then calculated by gas-particle drag and heat transfer.

An empirical relationship between critical velocity and particle material characteristics is used to determine deposition efficiency.
Apply Gasdynamic Analysis to Nozzle - Substrate Geometry

- Frictionless gas flow (except for particle interaction)
- Classical adiabatic, isentropic gasdynamic equations
- Particles do not influence gas conditions
- Shock standoff by Billig Approximation
- Normal component of the gas velocity linearly decreases to zero downstream of the shock
Nozzle Expansion
And Shock Calculations

\[
A_1 = A_2 = \frac{M_2}{M_1} \left\{ 1 + \left[ \frac{(\gamma - 1)}{2} M_1^2 \right] \right\}^{\frac{(\gamma + 1)}{2(\gamma - 1)}}
\]

Isentropic flow area/Mach number relationship

\[
\Delta = 0.143d_e \exp\left(3.24 / M_e^2 \right)
\]

Shock wave standoff distance*

\[
M^2 = \left[ \frac{M_e^2 + \frac{2}{\gamma - 1}}{\left( \frac{2\gamma}{\gamma - 1} \right) M_e^2 - 1} \right]
\]

Mach number after shock wave

Gas-Particle Relationships

Particle drag/velocity relationship

\[ m \frac{dV_p}{dt} = C_D \left( \frac{\pi}{8} \right) \rho_g d^2 \left( V_g - V_p \right)^2 \]

\[ C_D = \frac{24}{R_e} \left[ \left( 1 + 0.15 R_e^{0.687} \right) \left( 1 + e^{-0.427/M_p^{4.63} + 3.0/R_e^{0.88}} \right) \right] \]

Drag coefficient with rarefaction and compressibility effects*

Particle temperature/gas temperature relationship

\[ c_p \frac{dT_p}{dt} = \left( N_u k / d_p \right) \left( A_p / m \right) \left( T_g - T_p \right) \]

Nusselt number

\[ N_u = 2.0 + R_e^{0.5} P_r^{0.33} \]

Conditions of Calculations

Unless otherwise stated, all calculations are carried out for the following conditions:

• Nozzle area ratio = 4, Length = 0.1m
• Nitrogen accelerating gas
• 2.76 MPa, 673 degree K stagnation conditions
• Copper particles, inserted at 293 degree K
• Normal particle diameter distribution, SD = 4
### Calculation Detail

| \( M \) | M | T | P | \( V_p \) | \( \text{AV}^* \) | \( y \) | \( \mu \) | Vp | Tp | Tq | Vq | \( \text{AV}^* \) | Re | \( \eta \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0001 | 0.1 | 1.022 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 293 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.0007 | 0.1 | 1.0004 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.88E+00 | 3.88E+00 |
| 0.11 | M2 | 0.022 | 1.0008 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.02 | M2 | 0.002 | 1.0002 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.03 | M2 | 0.002 | 1.0002 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.04 | M2 | 0.002 | 1.0002 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.05 | M2 | 0.002 | 1.0002 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |
| 0.06 | M2 | 0.002 | 1.0002 | 672 | 397 | 1373 | 53 | 5.92 | 0.00241 | 0.00002 | 0 | 48 | 292 | 672 | 53 | 5.92 | 1.09E+01 | 0.010 | 3.08E+00 | 3.08E+00 |

**Gas & Particle Velocity & Temperature**

- Particle velocity
- Particle temp
- Nozzle exit
- Gas velocity
- Gas temp

### Diagram

- **Graph 1:** Gas & Particle Velocity & Temperature
  - Graph 2: Gas & Particle Velocity & Temperature

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0.00003 particle diameter m | 0.012 | 1.00007 | 672 | 397 | 1373 | 53 | 5.71 | 0.00239 | 0.00002 | 0 | 48 | 364 | 672 | 54 | 5.71 | 1.21E+01 | 0.010 | 3.02E+00 | 3.02E+00

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**Notes:**

- Calculation Table
- Diagrams of gas and particle velocity and temperature
Gas-Particle Flows
3 Micron Diameter Particle

![Graph showing velocity and temperature changes over length from nozzle entrance (m) for 3 micron diameter particle flows.](image-url)
Impact Velocity vs. Particle Diameter

Impact Velocity (m/s)

Particle Diameter (μm)

- 3g/cm³
- 6g/cm³
- 9g/cm³
9 g/cc Particle, Helium Gas

![Graph](image-url)

Y-axis: Impact Velocity, m/s
X-axis: Particle Diameter, micron
Deposition Calculations

The empirical relationship for the critical velocity is given by Assadi* as:

Critical Velocity = \(667 - 14\rho + 0.08T_m + 0.1\sigma_\mu - 0.4T_e\)


\(\rho\) = particle density  \(T_m\) = particle melting point

\(\sigma_\mu\) = particle UTS  \(T_e\) = particle exit temperature

All particles impacting the substrate with velocity larger than the critical velocity will deposit
Deposition Efficiency Calculation

1. Calculate particle impact velocities and temperatures vs. particle diameter.

2. Calculate critical velocities based on the particle impact temperatures.

3. Find the range of particle sizes that has an impact velocity greater than the critical velocity.

4. Given the MMD and the SD of the powder, integrate the normal particle size distribution from the smaller diameter to the larger diameter.
Critical Particle Diameters

![Graph showing critical particle diameters and velocities.](image)

- Critical particle diameters deposit these particles.

- The graph shows the relationship between particle diameter (µm) and velocity (m/s), with critical velocity indicated for deposition.
Effect of Particle Diameter on Deposition Efficiency

![Graph showing the effect of particle diameter on deposition efficiency with and without shock. The graph plots deposition efficiency (%) against distribution MMD (µm). The red line represents no shock, and the blue line represents with shock. The efficiency decreases as the MMD increases.]
Conclusions

• Decreasing particle diameter results in improved deposition only to a point.

• This point is reached at approximately 5 microns, as smaller particles are more readily slowed in the flow downstream of the bow shock.

• This effect can result in a reduction of the deposition efficiency from 90% to 50% when spraying powders with an MMD of 8 versus an MMD of 2.
Questions?